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# Local and Global Risk Aversion

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In response to Horowitz's (1998) comments on their 1996 paper, Prakash et al. (1998) present a cubic utility function which, according to them, is risk-averse and yet allows a preference for unfair gambles. But that utility function is only locally risk-averse, and the wealth levels involved in the preferred gamble exceed the risk-averse interval. Since global risk-aversion implies no unfair gambles will ever be preferred, some form of non-globally risk-averse utility function has to be assumed in order to justify a preference for unfair gambles. A brief discussion of a few alternatives is given to show the weaknesses of non-globally risk-averse utility functions.

## Introduction

In Prakash et al. (1996), the authors claimed that a risk-averse decision maker may prefer an unfair gamble if he / she prefers positive skewness. Assuming the risk-aversion to be global, Horowitz (1998) showed that it cannot happen. In response, Prakash et al. (1998) present the cubic utility function

$$U(x) = c_1x + c_2x^2 + c_3x^3,$$

where

$c_2 < 0$
$c_3 > 0$ [It is also assumed that
$U'(x) = c_1 + 2c_2x + 3c_3x^2 > 0$ for all $x$.]

They go on to show the following.

This $U(x)$ is risk-averse for $x < -c_2 / 3c_3$. Now, if an unfair lottery of winning a prize amount of $\theta$ with probability $p$ with $p\theta < 1$ is available at a price of 1, the decision maker would buy the lottery if the initial wealth $x$ is greater than $(1-\theta) / 3 - c_2 / 3c_3$. Thus, according to the authors, a risk-averse decision maker prefers an unfair gamble.

But from any initial wealth greater than $(1-\theta) / 3 - c_2 / 3c_3$, the terminal wealth when the prize $\theta$ is won exceeds $-c_2 / 3c_3$ where the decision maker is not risk-averse. It is therefore no surprise that the decision maker prefers the unfair gamble.

## Local and Global Risk Aversion

Jensen's inequality implies that a globally risk-averse decision maker will never accept an unfair gamble. If we are to find a rationale for accepting unfair gambles within the expected utility framework, we have to abandon global risk aversion. Let us look at a few non-globally risk-averse utility functions.

A plot of the cubic $U(x)$ above will look like Figure 1 with a concave section on the left and a convex section on the right. The corresponding decision maker with initial wealth $x_0$ will show preference for positive skewness, prefer unfair gambles and buy insurance against losses.

Suppose we add a convex section to the above utility curve on the left as shown in Figure 2. In this case, the decision maker will show preference for not only positive skewness by engaging in unfair gambles involving huge gains but also negative skewness by engaging in unfair gambles involving huge losses. The preference for positive skewness emphasized in Prakash et al. (1996) is a particular case and not general.

But in both of the above cases, the unbounded convex section of the utility curve on the right makes the decision maker a reckless gambler. He / she will engage in too many unfair gambles, and will be ruined. An unbounded convex section on the right is therefore undesirable. The second case is worse because, in addition, it has a (wealth-unbounded) convex section on the left which is also undesirable. The decision maker will recklessly undertake unfair gambles involving huge losses. In order to

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avoid reckless gambling on the right and the left, convex sections of a utility curve should be confined to the middle.

Friedman and Savage (1948, 1952) use a utility function that looks like Figure 3. Although this utility function avoids reckless gambling on the right and the left, it is not without trouble. When the current wealth is in the convex region, the decision maker will gamble (and buy insurance against large losses), but once the wealth level falls below or rises above the convex region due to repeated gambling, the decision maker will cease to gamble. That is not the predominantly observed gambling behavior in real life. In order to continue gambling, the utility function should "shift" so that the current wealth continues to be in the convex region. Thus, in order to explain observed behavior, not only do we need to assume an odd-shaped utility function but we also need to assume that it shifts. To most theorists, this is unsatisfactory.

One may assume a utility function with several convex and concave sections as in Figure 4. But it would not escape the criticisms above. This inability of the expected utility theory to explain commonly observed risk taking behaviors is indeed the main motivation for non-expected utility theories.

The above criticisms of utility functions that are not globally risk-averse are not new and can be found in numerous places in the literature. Quiggin (1993) gives a concise and excellent account of these issues with the expected utility theory, and then expounds the leading non-expected utility theory, namely, the Rank Dependent Expected Utility theory.

## Concluding Remarks

Many of the commonly observed risk taking behaviors do not fit the expected utility theory, especially with globally risk-averse utility functions. Preference for unfair gambles, such as lotteries, is one such phenomenon. Suppose we abandon global risk aversion and look for a suitable utility function with convex and concave sections. Any such function will have the weakness of either allowing reckless gambling, or requiring the assumption of shifts. These issues and others have motivated the development of non-expected utility theories.

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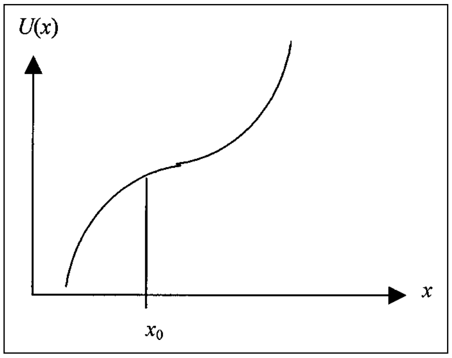


***Figure 1.***

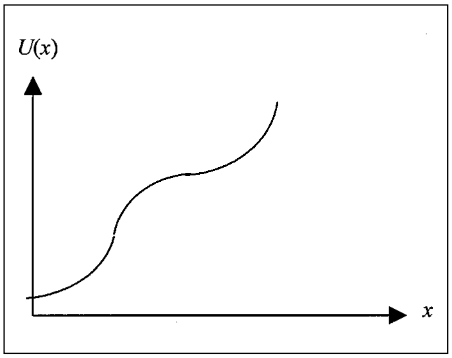


***Figure 2.***

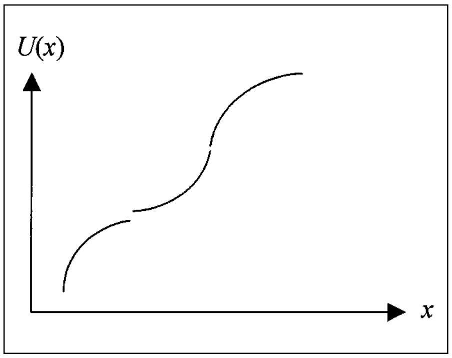


***Figure 3.***

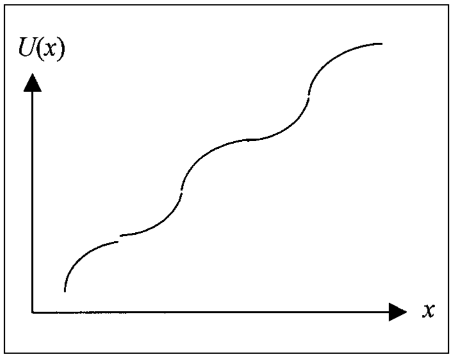


***Figure 4.***