Dynamic Postoptimal Analysis

Rick Hesse, Feature Editor

The Solver included in Microsoft's Excel is indeed a handy tool for solving linear and nonlinear models. Not only has the speed and efficiency of computers increased in the ten-year life span of this product by Frontline Systems, but so has the speed and efficiency of the Solver itself. Having worked with MPS and MPSX in the '60s on mainframe computers, it is a delight to be able to solve decent-sized LP models on a spreadsheet platform rather than have to always solve two-variable problems for classroom exercises. However, for this column I want to present a small model (so it will fit in the column!) that illustrates the power of the Solver to do some good what-if analysis, which is such a rich part of LP modeling. There is so much more information to be gained after the original solution that is now enabled by features of the solver, which will allow variables to be on both the left and right hand sides of the constraints.

Basic Model

TRANSCO produces two types of antennas and is limited by four production processes, as shown in the optimal solution in Figure 1, with the time per unit and cost per minute in B4:F7. I always like to put the data at the top of the spreadsheet and then the model below, pointing to the data, so that any change in the data will automatically change the model. The only formula other than "pointing" to the data is found in D11 and is = SUMPRODUCT(B11:C11, $B$15:$C$15) and copied to D12:D13 and D17:D20.

	A	B	C	D	E	F
1	TRANSCO					
2	TRANSCO	Production Time (minutes)		Total		Cost/
3		Long Range	Microwave	Minutes		Minute
4	Shearer	33	23	1,200		$2.080
5	Drill & Deburr	28	20	1,000		$1.540
6	Assembly	25	15	800		$1.700
7	Inspection	32	25	1,100		$1.960
8	Min Orders	5	8			
9						
10		Long Range	Microwave			
11	Costs	$216.98	$153.14	$7,244.11		Cost
12	Revenue	$350.00	$250.00	$11,724.14		Revenue
13	Maximize Profit	$133.02	$96.86	$4,480.03	<=OPTIMIZE	
14		Long Range	Microwave			
15	DECISIONS =>	24.14	13.10	Amount Used		Need/Available
16	Subject to:	Long Range	Microwave	CONSTRAINTS		Resources
17	Shearer	33	23	1,097.93	≤	1,200.00
18	Drill & Deburr	28	20	937.93	≤	1,000.00
19	Assembly	25	15	800.00	<	800.00
20	Inspection	32	25	1,100.00	<	1,100.00

Figure 1: Base case for TRANSCO products.

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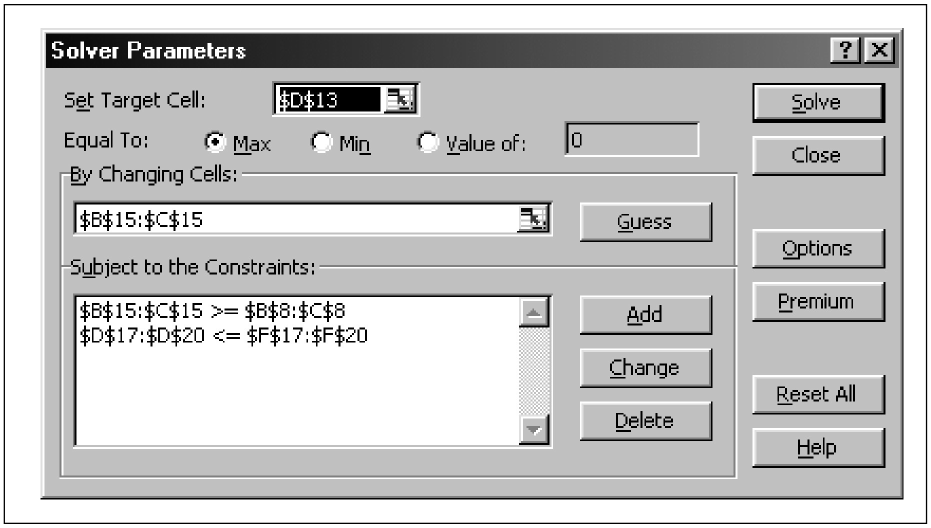


Figure 2: Solver setup for base case.

Postoptimal Report

	Decision Name	Final Value	Marginal Value	Lower Limit	Original Value	Upper Limit
	Long Range	24.14	$0.00	$123.98	$133.02	$161.43
	Microwave	13.10	$0.00	$79.81	$96.86	$103.92
	Resource Name	**Final Value**	**Marginal Value**	**Lower Limit**	**Original Value**	**Upper Limit**
≤	Shearer	1,097.93	$0.00	1,097.93	1,200.00	1E+30
≤	Drill & Deburr	937.93	$0.00	937.93	1,000.00	1E+30
≤	Assembly	800.00	$1.56	689.00	800.00	823.13
≤	Inspection	1,100.00	$2.94	1,070.40	1,100.00	1,212.50

Figure 3: Postoptimal report for base case.

One Extra Minute	Cost	MV	ROR
Assembly	$1.700	$1.558	91.7%
Inspection	$1.960	$2.939	150.0%

Figure 4: Rate of return for one extra minute.

The Solver setup is given in Figure 2 and the options checked are Assume Linear Model and Assume Non-Negative. Note that the minimums for production are given in the constraint $B$15:$C$15 >= $B$8:$C$8.

When the Excel Sensitivity Report is saved, I usually have students write their own Postoptimal Report from the Excel Report to look like Figure 3.

We notice immediately that two of the processes have run out of time, and therefore have a positive marginal value for extra time. Every extra minute of Inspection time at regular price ($1.96/minute) would make $2.94 more in profit, while every extra minute of Assembly at the regular price of $1.70 would yield $1.56 in profit. It is usually at this point that students insist that no more Assembly time should be purchased because you would lose money! By rerunning the model with 801 minutes of Assembly time, an object lesson in what marginal values really mean can be quickly made. A simple table such as in Figure 4 shows the value of an extra minute for each process and the rate of return (ROR) as the profit divided by the cost.

This shows that if TRANSCO could get a few dollars more for extra time, the company should buy Inspection minutes first (assuming availability) because the ROR is greater (*not* because the marginal value is larger!). It is here that another lesson can be emphasized—the model doesn't need both Assembly and Inspection minutes to make more profit. This same lesson was also shown when the model was rerun with 801 minutes of Assembly time. The Postoptimal Report shows that Inspection could use 112.5 more minutes at $1.96 cost, which would be $220.50.

Surcharge

But usually extra time is not available at the same rate—there may be issues of overtime, union rules, availability of qualified workers, and so forth. Overtime may be at the rate of time and a half (or a 50% surcharge) or double time, and for raw materials, you might have to pay a premium for overnight shipping or a premium for more storage. Shown in Figure 5 is an extra time model that assumes that there is a surcharge of 50% for extra time on any of the processes, with a maximum of three hours available on each. Four extra variables are added in G17:G20, with limits for overtime for each machine in H17:H20. There is also an added constraint on the expenditure for extra time (regular plus overtime) in G10, with the costs broken down in F8 (surcharge) and F9 (regular time cost).

The right hand side (RHS) of the constraints now have the formula in cell F17 =D4+G17 copied down to G20, which is the original allowable minutes plus the extra minutes. The Solver setup in Figure 6 shows the added constraints and variables. Both the Cost in D11 and Profit in D13 have cell F8 subtracted from the SUMPRODUCT. The two sets of constraints that have been added are for the upper limit on extra time ($G$17:$G$20 <= $H$17:$H$20) and the budget constraint ($F$10 <= $G$10). The regular cost in F9 is the sumproduct of the

	A	B	C	D	E	F	G	H
8				Surcharge			Surcharge	
9				Reg Cost		$333.33	50%	
10		Long Range	Microwave	Total		$500.00	$500.00	
11	Costs	$216.98	$153.14	$8,014.89		Cost		
12	Revenue	$350.00	$250.00	$12,776.96		Revenue		
13	Maximize Profit	$133.02	$96.86	$4,762.07	<=OPTIMIZE			
14		Long Range	Microwave					
15						Need/Available	Extra	
16	Subject to:	Long Range	Microwave	CONSTRAINTS		Resources	Minutes	Max
17	Shearer	33	23	1,182.16	≤	1,200.00	0.00	180
18	Drill & Deburr	28	20	1,022.16	≤	1,022.16	22.16	180
19	Assembly	25	15	800.00	≤	800.00	0.00	180
20	Inspection	32	25	1,252.66	≤	1,252.66	152.66	180

Figure 5: Extra time model with surcharge with $500 budget.

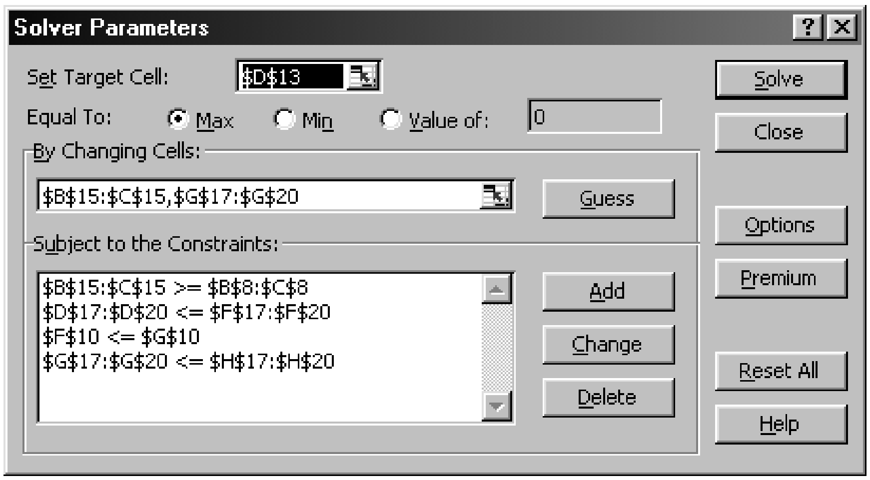


Figure 6: Solver setup for extra time TRANSCO model.

	Resource Name	Final Value	Marginal Value	Lower Limit	Original Value	Upper Limit
≤	Shearer	1,182.16	$0.00	-17.84	0.00	1E+30
≤	Drill & Deburr	1,022.16	$1.61	-226.27	0.00	31.76
≤	Assembly	800.00	$0.89	-17.08	0.00	36.51
≤	Inspection	1,252.66	$2.05	-57.57	0.00	46.36
≤	Extra Cost	$500.00	$0.36	$330.75	$500.00	$615.23

Figure 7: Bottom part of postoptimal report for extra time TRANSCO model.

original costs and extra minutes, and the surcharge is G9*F9. The sum of F8 and F9 in F10 is bounded by G10.

Postoptimal Surprise

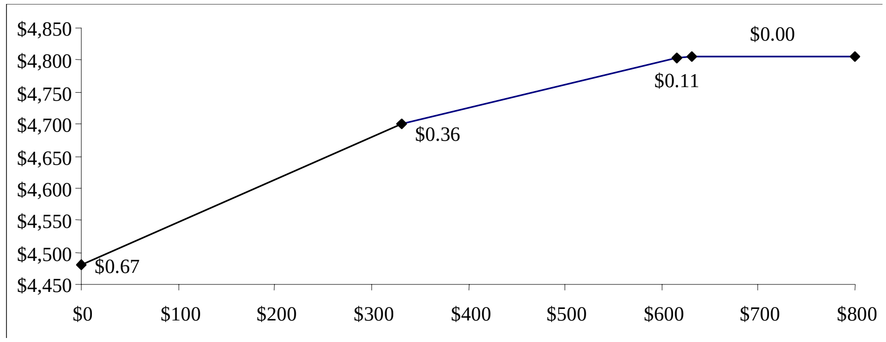
The biggest surprise about the results of the extra time model, is that given some extra money, TRANSCO ends up buying not only the extra Inspection time, but Drill time rather than Assembly time. This is not what we might have guessed just looking at the Postoptimal Report, which still contains some very valuable information. Because we are able to add variables to the RHS of the constraints, we are able to let the Solver show what would happen if we had more money and how it would be spent. By making up a Postoptimal Report for the extra time model (with only the constraint report shown in Figure 7), we can see that the lower limit of adding extra money is $330.75, which is simply the original cost of all the Inspection time ($220.50) plus the surcharge of 50%.

Because the cells of the RHS time constraints have variables added, the original RHS is now seen as 0.00. The marginal values tell us that Assembly time is now third in priority for extra minutes rather than second. If we run the model at the upper limit break point for Extra Cost and find the other break points, we can get the summary table and graph shown in Figure 8. This table and graph provide valuable managerial information about the processes. In the "old" days of linear programming, we would have had to use parametric analysis, adding fixed amounts of overtime rather than as needed.

The table shows the range of Extra Capital, from $0.00 (base case) to $630.98. Past that amount extra money doesn't help due to the limitations of extra Assembly time. We see that extra time is first used only for the Inspection and that the model produces less Long Range and more Microwaves than the base case and every extra dollar invested earns $0.67 up to $330.75 of extra costs. For the next $300, more Inspection and Drill time is purchased, with the marginal value decreasing to $0.36 for each dollar spent. Finally, for a short while, extra Assembly time is bought until the model has reached the limits of making it worthwhile to buy extra time. In fact, less than five minutes of Assembly time is actually purchased, even though our base case postoptimal report showed that more Assembly time might be worthwhile.

Conclusion

When all is said and done, this little exercise shows the power of spreadsheet modeling, the solver, and some good analysis. It demonstrates that modeling is not just finding a number as an answer, but that there is a wealth of information in the base solution to the model, the Postoptimal Report, and then some dynamic postoptimal analysis after that. ■



$ Extra Capital	Long Range	Micro-wave	Extra Shearer	Extra Drill	Extra Assembly	Extra Inspection	Profit	Marginal Value
$0.00	24.14	13.10	0.00	0.00	0.00	0.00	$4,480.03	$0.67
$330.75	12.50	32.50	0.00	0.00	0.00	112.50	$4,700.45	$0.36
$615.23	5.52	44.14	0.00	37.24	0.00	180.00	$4,804.03	$0.11
$630.98	6.29	43.15	0.00	39.10	4.49	180.00	$4,805.78	$0.00
$800.00	6.29	43.15	0.00	39.10	4.49	180.00	$4,805.78	

Figure 8: Summary table and graph of extra time for TRANSCO.

NAMES IN THE NEWS

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Mohan V. Tatikonda, University of North Carolina at Chapel Hill, and **Stephen R. Rosenthal**, Boston University, were awarded the Chan K. Hahn prize for best paper in the Operations Management Division of the 1999 Academy of Management Conference. A multi-tier double-blind review process was conducted among all submissions to the division. The paper's title is "Successful Execution of Product Development Projects." The paper's findings provide insight into project management practices firms can employ to bring new products to market faster. Based on analysis of 120 new product development projects at high-tech firms, the paper finds that risky development projects are best managed through a combination of structured and informal management processes.

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