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Should We Teach Sensitivity Analysis Report?

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In the July 2001 issue of *Decision Line*, Professor Rick Hesse alerts us in this column that the sensitivity analysis report of a linear programming problem must be read carefully when there is a high degree of degeneracy in the solution. Using an assignment problem as example, he demonstrates that the range for the objective function coefficient of a variable, found in the sensitivity analysis report, can be exceeded without changing the optimal solution. He also describes the use of the Solver Table to find when exactly the optimal solution does change. I would like to place his message in a sharper focus that highlights the significance of bases and brings out some larger issues of pedagogy.

Degeneracy and Sensitivity Analysis

What is the exact significance of the range of a variable found in the sensitivity analysis report? It is true that within the range, the optimal basis will not change (fact 1) , and the optimal solution will not change (fact 2) .

It is also true that when the range is exceeded, the optimal basis will change (fact 3), and the optimal solution *may or may not* change (fact 4).

Furthermore, since the simplex algorithm finds only the basic solutions, the basis has to change if the optimal solution has to change. Thus, we have the range in which the optimal solution does not change is wider than or equal to the range in which the basis does not change (fact 5).

Hesse directs our attention to facts 3, 4, and 5 in his article. Just to reacquaint the reader with these facts, I shall use the following quick example. Consider the problem:

$$\text{Maximize } 3x + y + z$$

subject to

$$x + y \leq 10$$
$$x + z \leq 10$$
$$x, y, z \geq 0.$$

The obvious optimal solution is $x = 10$, $y = z = 0$. Since the problem is symmetric about y and z, one would expect them to have the same Reduced Cost (I am using the Excel Solver terminology) and their objective coefficients to have the same Allowable Decrease. But due to the degeneracy, the sensitivity report shows for y a Reduced Cost of 0 and an Allowable Decrease of 1. For z, it shows a Reduced Cost of –1 and an Allowable Decrease of infinity. (Depending upon how the problem is entered and initialized the values for y and z may be switched.) Consider the Allowable Decrease of 1 for the objective coefficient of y. It is obvious that the optimal solution will not change if the coefficient decreases by more than 1. Indeed, any decrease only favors keeping y equal to 0.

Similar comments can also be made about the reduced cost of 0 for y. A slight increase, say, by 0.1, in the coefficient of y is not going to change the solution.

The facts do not stop here. They extend to the Shadow Prices and RHS ranges as well. To see it, in the problem above add a third constraint identical to the second. Then solve the problem and obtain the sensitivity analysis report. The two identical constraints will have different shadow prices and ranges. Facts similar to the five listed above apply to these Shadow Prices and ranges as well. To be explicit, I shall enumerate them.

Within the reported range of the RHS of a constraint, the optimal basis will not change (fact 6), and the Shadow Price of the constraint will not change (fact 7).

Outside the range, the optimal basis will change (fact 8), and the Shadow Price of the constraint may or may not change (fact 9).

Because a change in the optimal solution requires a change in the optimal basis, the range in which the Shadow Price does not change will be wider than or equal to

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the range in which the optimal basis does not change (fact 10). All these bring up an important issue in pedagogy.

Implications on Pedagogy

In the classroom, instructors have to be careful while explaining the ranges mentioned in sensitivity analysis reports. Especially, fact 4 above should be worded properly. (At one point in his article, Hesse himself slips and writes, “exceeds the upper bound, the solution *should* change.”) Evidently, fact 4 should be taught in conjunction with fact 3. But this necessitates teaching the students what a basis is. For those of us who have given up teaching the simplex algorithm in favor of teaching the use of the Solver, this could be a formidable task. What should we do? My answer is:

> If one teaches the use of the Solver rather than the simplex algorithm, then one should not teach anything about the sensitivity analysis report. Instead, one should teach the students to re-solve a problem with changed data to find the effects of changes in data on the results.

Why Re-Solving a Problem Is Better

The following reasons explain why re-solving a problem is better than using a sensitivity analysis report.

1. A sensitivity analysis report gives only limited information. For instance, a Shadow Price shows the impact of a change in an RHS (within allowable limits) on the objective function value, but nothing in the report shows how the optimal solution would change. For all practical purposes, one needs to know the new optimal solution.

2. The applicability of the information in a sensitivity analysis report is limited. For instance, a Shadow Price may not be valid when the corresponding RHS range is exceeded. Even a greater limitation is the fact that every value in a sensitivity analysis report can be invalid, when several changes in the data, all within respective ranges but sufficiently large, occur simultaneously. In practice, many changes do occur simultaneously.

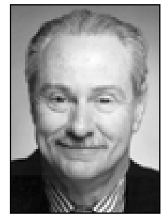
3. Every information in a sensitivity analysis report can be obtained by re-solving the problem. Thus, nothing is lost by not having a sensitivity analysis report, if one can re-solve a problem.

To find the impact of a change in data, why not simply re-solve the problem and get the full impact of the change under all circumstances? By re-solving, we would get the new optimal solution, and the new optimal value of the objective function. The new solution tells us what to implement and the new value of the objective function tells us what the net gain or loss is. These details should be enough to answer any practical question. Note that Hesse’s solution to find the “true range” of an objective function coefficient was to use the Solver Table, which is nothing but re-solving the problem several times.

Concluding Remarks

The sensitivity analysis report is a great convenience if the problem is too large to re-solve. Given today’s computers, a vast majority of the problems that a manager faces are not too large to re-solve within a reasonable amount of time (say, within several minutes). Re-solving a problem tells us everything needed for practical decision making while a sensitivity report may not. Furthermore, a true understanding of the sensitivity analysis report requires a thorough knowledge of the meaning and the applicability of the reported values. The concept of basis, which is necessary for understanding a sensitivity analysis report thoroughly, is difficult to teach if the simplex algorithm is not taught. Thus, it is better to teach students how to re-solve a problem than to teach them how to use a sensitivity analysis report.

Finally, I wish to clarify that I am not against teaching sensitivity analysis together with the simplex algorithm. ■



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