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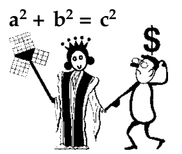
Pure Mathematics and the Weiszfeld Algorithm

by Andrew Vazsonyi, Feature Editor



THE MAY ISSUE of *Computers & Operations Research* has an article "On the circle closest to a set of points." Interestingly enough, the article mentions the Weiszfeld algorithm. Why should this catch my attention? Well, it so happens that I am the discoverer of the Weiszfeld algorithm. Let me explain.

In the 1960s a management scientist colleague called me and asked for a pointer on convex programming.

"But I know nothing about it," I said.

"Then I suggest you read Chapter Seven, "Convex Programming" in your book *Scientific Programming in Business and Industry*," he replied.

For a moment, I felt embarrassed because I couldn't remember what was in my own book, but I didn't say anything about it.

We chatted for a while longer and before he hung up, he said, "I'm glad I could get better acquainted with the guy who has done such pioneering work in location theory."

Again it was my turn to feel absent-minded. "What's that?"

"Suppose we have a bunch of factories and want to build a warehouse in a central location so the transportation costs are minimized."

"That's a tough nonlinear programming problem," I said. "But I never worked on it." Actually, I began to feel some relief as it became more obvious that this was a case of mistaken identity.

"I must be confused," he replied.

There the matter rested until a few months later when I ran across a reference to a paper in a production control article: "New developments on the Weiszfeld algorithm."

Weiszfeld is a very unusual name. To my knowledge, there was only one family in Hungary with that name – mine! So I ordered a reprint, and to my amazement I learned that my paper, published in the distant past, had used a "long-step algorithm" (which I have never heard of!) and was considered a basically new approach to the location problem.

After reading more, I discovered that a well-known mathematician, Harold W. Kuhn of Princeton University, had given a talk in Budapest on his discovery of an algorithm to solve the location problem. After the talk, a former colleague of mine walked to the blackboard and wrote in big letters: VAZSONYI."

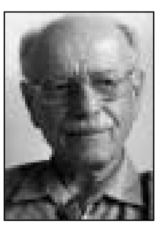
"Who is that?" Kuhn asked.

"The name of the Hungarian mathematician who discovered your algorithm thirty years ago," my old friend said. "He lives in the United States but published his revolutionary approach under the name Weiszfeld."

My paper "*Sur le point pour lequel les sommes des distances de n points donnés et minimum*" was actually published in Japan in 1937 in the *Tôhoku Mathematical Journal* (volume 43, pages 355—386) under the name of Endre Weiszfeld. This was a few years before I changed my name to Vazsonyi.

Unbeknownst to me, the article later became a classic in the mathematics of location analysis. Today the essence of the paper is referred to as the "Weiszfeld algorithm, which finds the point that minimizes the sum of the distances from N given points."

Location analysis goes back to the influential book by the German industrial author Alfred Weber (1909). Here's a simple example of location analysis. Let's say a firm wants to locate a factory so that the transportation cost to the warehouses is a minimum. Mathematically speaking, the problems is to find the unknown point that minimizes the sum of the distances from N given points.



Andrew Vazsonyi *is an internationally recognized author, researcher and educator. He is the author of over 70 technical articles, and seven textbooks, in English, German, Spanish, French, Russian, Japanese and Hungarian. Dr. Vazsonyi received a Ph.D. from the University of Budapest. He is currently an emeritus professor at San Francisco University and has 20 years of teaching experience. Prior to becoming an educator, he served for 25 years in industrial positions. These days he focuses on books and articles that apply Microsoft Excel and VBA to production and operations management.*

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Mathematicians have been fascinated by the problem ever since Pierre de Fermat (1601-1665), author of the celebrated "Fermat's Last Theorem" (proven only in 1994 by Andrew Wiles, an English mathematician), solved the problem for three points, that is, for a triangle. However, for more than three points there has not been any progress until the 1960s, when the issue of the location of site facilities in diverse environments became important and computers allowed the practical execution of algorithms.

In the 1930s, when I became interested in the problem at the age of 16, I did not have in mind to actually calculate the coordinates of the unknown point. However, I was searching for a theorem to prove. So I hit on the bizarre idea of searching for an iterative process to find the point, and I did find the process. (The word *algorithm* was unknown to me and to most mathematicians.) Here was a theorem to prove: Does the process converge? Yes, I proved that it does. And that was what the Tôhoku article was about. Unfortunately, other mathematicians weren't interested at the time and the paper was forgotten until the 1960s when it was rediscovered and recognized as the key algorithm needed for location theory. Many computer programs were developed to do the actual calculations. Thus a pure mathematical effort, a gleam in the eye, became a practical managerial tool.

The other day I searched on Google for the "Weiszfeld algorithm" and received over 160 hits, mostly referring to English articles, though some were in German and Spanish. I reread the article and was reminded of what my fellow countryman (distinguished Hungarian mathematician and author of the best seller *How to Solve It*) George Polya told me in Palo Alto in 1983 when he was 96. "I cannot create anymore, so I am just studying my old articles and shaking my head: I used to be a pretty clever mathematician."

To my great chagrin, nobody knows that Weiszfeld is around and kicking and Vazsonyi $_{2002}$ = Weiszfeld $_{1934}$. ■

Decision Sciences Institute Budget Summary FY 2002-2003

July 1, 2002-June 30, 2003

Revenues summary		
Publications	$ 81,371	
Membership Revenues	192,683	
Electronic Publishing	0	
Convention	324,873	
Total revenues		$598,927
Expenses summary		
Publications	$149,067	
Member Services	198,764	
Electronic Publishing	17,953	
Convention	228,280	
Total expenses		$594,064
Net Revenue Over (Under) Expenses		4,863
Plus Depreciation Expense (Not a cash expense)		9,846
Net Revenue Over (Under) Cash Expenses		$14,709

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