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Tickling the Solver with Backwards Bounds Constraints

by Rick Hesse, Feature Editor, Pepperdine University

Frontline Systems' Solver, which is included in Excel, has a bounded variable simplex code. This means that for linear programming problems, variables that have bounds don't need constraints written on the spreadsheet, but these bounds can simply be added to the Solver setup.

Unfortunately, when this is done, the Excel Sensitivity Report does not contain the range for the marginal value (MV) of each bounded variable, just the marginal value (reduced cost, shadow price, LaGrange multiplier, partial derivative, reduced gradient—call it what you will).

	A	B	C	D	E	F	G	H	I	
1	Metal Tables								Bounds constraints in Spreadsheet Model only	
2	Metal304	UNIT DATA							Solver Parameters	
3	Base Model	Large	Small	Computer			Avail Hours	Set Target Cell: \$E\$10		
4	Drill (min)	14	10	8			40	Equal To: Max Min		
5	Press (min)	15	8	12			30	By Changing Variable Cells: \$B\$12:\$D\$12		
6	Assembly (min)	20	26	25			70	Subject to the Constraints:		
7	Maximum	50	150	40	Conversion			\$E\$14:\$E\$19 <= \$G\$14:\$G\$19		
8	Profit	\$36.00	\$25.20	\$44.15	60	min/hr		Solver Options		
9	Contributions								Max Time: 100 second	
10	Profit	\$1,364.87	\$2,366.61	\$1,766.00	\$5,497.48	<= OPTIMIZE		Iterations: 100		
11		Large	Small	Computer			Total Used	Precision: 0.000001		
12	DECISIONS =>	37.91	93.91	40.00			LHS	Tolerance: 0.05		
13	Subject to:	Large	Small	Computer			RHS	Convergence: 0.0001		
14	Drill (min)	530.78	939.13	320.00	1,789.91	<=	2,400	<input checked="" type="checkbox"/> Assume Linear Model		
15	Press (min)	568.70	751.30	480.00	1,800.00	<=	1,800	<input checked="" type="checkbox"/> Assume Non-Negative		
16	Assembly (min)	758.26	2,441.74	1,000.00	4,200.00	<=	4,200			
17	Max Large	37.91			37.91	<=	50			
18	Max Small		93.91		93.91	<=	150			
19	Max Computer			40.00	40.00	<=	40			

Figure 1: Simple LP with regular bounds solver setup.



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applied math and computer science. Dr. Hesse is the author of *Managerial Spreadsheet Modeling & Analysis and Applied Management Science: A Quick & Dirty Approach* (with Gene Woolsey), articles in numerous journals, and software for personal computers. Rick was the first professor to be awarded the Outstanding Civilian Service Medal by the Department of the Army at West Point in 1982, and was the winner of the Decision Sciences Institute's Innovative Instructional Award in 1981.

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Adjustable Cells						
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$12	DECISIONS => Large	37.91	0.00	36	11.25	16.6153846
\$C\$12	DECISIONS => Small	93.91	0.00	25.2	20.15185	6
\$D\$12	DECISIONS => Computer	40.00	0.00	44.15	1E+30	11.8282609

Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$14	Drill (min) LHS	1,789.91	0.00	2400	1E+30	610.086957
\$E\$15	Press (min) LHS	1,800.00	1.88	1800	106.9231	335.384615
\$E\$16	Assembly (min) LHS	4,200.00	0.39	4200	860	347.5
\$E\$17	Max Large LHS	37.91	0.00	50	1E+30	12.0869565
\$E\$18	Max Small LHS	93.91	0.00	150	1E+30	56.0869565
\$E\$19	Max Computer LHS	40.00	11.83	40	77.85714	24.8214286

Figure 2: Excel sensitivity report for simple LP Model with bounds in solver setup.

Regular LP Model

Consider a simple LP model with four regular constraints and upper bounds on the three variables written on the spreadsheet. The data and model solution are shown in Figure 1.

E10: =SUM(B10:D10) copy to E14:E19
 G14: =G4*\$E\$8 copy to G15:G16
 B14: =B4*B\$12 copy to B14:D16
 B17: = B\$12 copy to C18, D19
 G17: = B7
 G18: =C7
 G19: =D7

The shading in cells G15:G16 and G19 is done by Conditional Formatting to indicate binding constraints. The Solver Setup is saved by using <Alt>+<PrntScr> on most computers, which captures the active or front window (but you must close the Solver before being able to save the picture to the spreadsheet). Then you can use the spreadsheet to “cut & paste” the Solver pictures down to size. The optimal solution to this problem shows that Computer Tables are at their upper limit, and the Excel Sensitivity Report is shown in Figure 2 (with some reformatting to make it easier to read).

The reduced cost in the last line (marginal profit, or more generically, marginal value – MV) is \$11.83 for every extra table that production would allow.

Bounds Constraints in Solver Only

To allow the LP model to take less room on the spreadsheet, it is possible to add the bounds constraints in the Solver only and eliminate rows 17–19 on the spreadsheet. The only problem is that although the Excel Sensitivity Report gives the correct marginal value in the Reduced Cost column as shown in Figure 3, it does not inform us if this is for just a fraction of one more table, five more tables, or 100 more tables. This is because the constraint for Computer Tables was not written directly in the spreadsheet model. In this case we would have to change the limit from 40 to 41 or 45 or 140 and rerun the Solver to determine the limit above 40, and do something in similar fashion to determine the lower bound. Note that if you do not have the Educational Premium Solver (comes free with most MS/OR college textbooks)

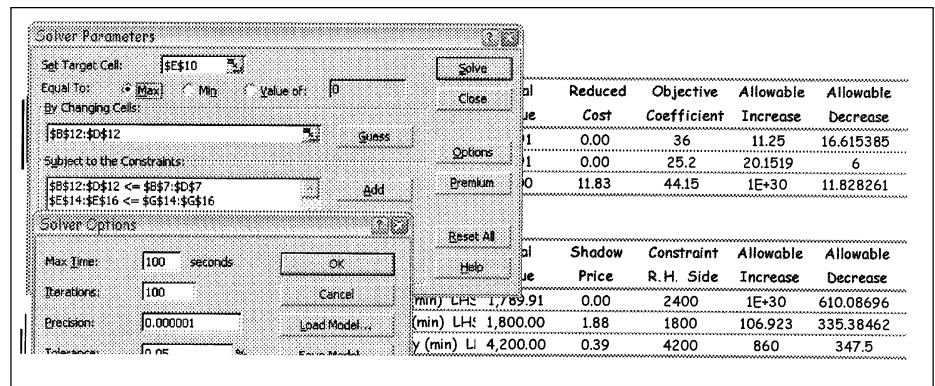


Figure 3: Excel sensitivity report with bounds constraints in solver only.

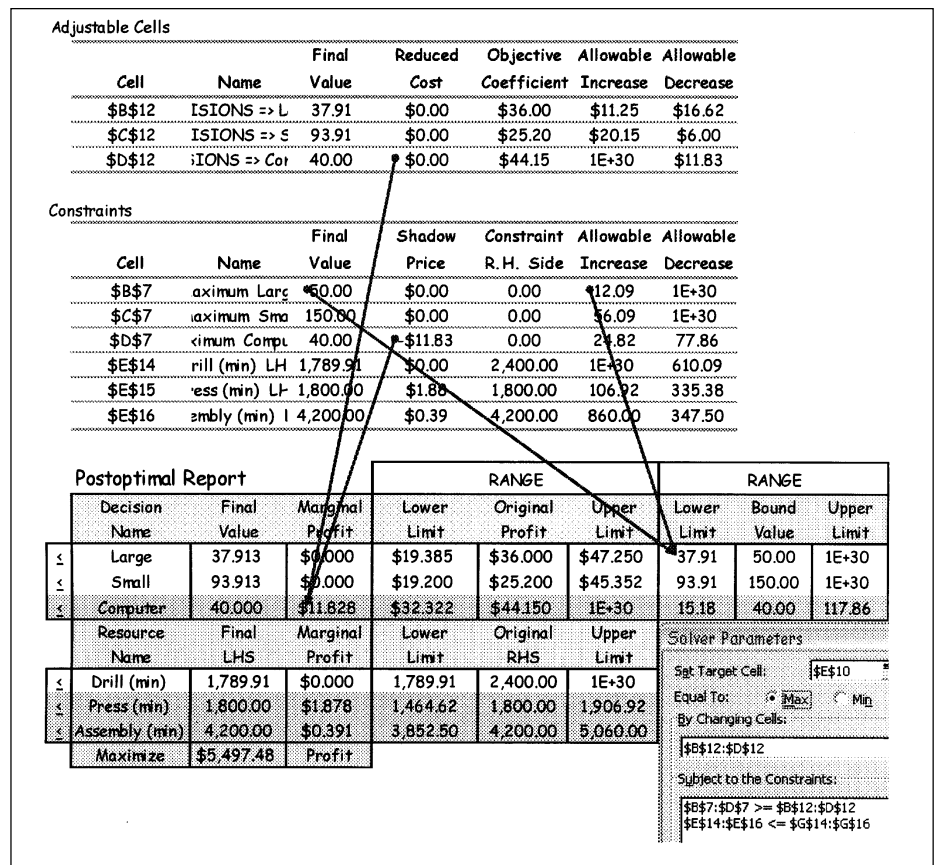


Figure 4: Excel sensitivity report with backwards bounds.

then your Excel Sensitivity Report may order the constraints differently.

Good Mistakes

I usually have my students include bounds on the variables in the Solver setup only, and simply lament that we can’t determine the range of these MVs from the Excel Sen-

sitivity Report. Then last semester, I had a student accidentally write the bounds “backwards,” as shown in Figure 4, yielding the same optimal solution, but with a surprising Excel Sensitivity Report. By “backwards” I mean that the bounds for the variables are written as Maximums >= Variables instead of Variables <= Maximums. This induces the Solver Sensitivity

Report to treat the Bounds in the Solver as if they appeared on the spreadsheet, and gives out valuable information, but in a “backwards” or “topsy-turvy” manner. The good folks at Frontline Systems don’t recommend doing this, because it requires some careful reading (or rewriting) of the Sensitivity Report; otherwise users may be misled by the results. But it is worthwhile learning how to obtain this information in a careful manner. It is also important that you either have the Premium Educational Solver installed (which automatically sorts the Solver constraints in row order) or pay careful attention to the order of the constraints in the Excel Sensitivity Report.

Notice in Figure 4 that the Excel Sensitivity Report has three extra rows added in the Constraints section that Figure 3 did not have. These three rows have the information we had in Figure 2, but all jumbled up. The marginal value for each variable is the Reduced Cost in the top portion minus the Shadow Price in the bottom portion of the Excel Sensitivity Report. The Final Value in the bottom portion of the Excel Sensitivity Report is really the Maximum bound, the Allowable Increase is really the decrease, and the Allowable Decrease is really the increase. Confusing? Yes, and that’s why Frontline judiciously doesn’t recommend doing this. But this does yield the information about the range for Computer Tables for which the marginal profit of \$11.83 is good—from 15.18→117.86 Computer Tables. I prefer to have my students paste this report on the model sheet and then build a more readable Postoptimal Report below it, as shown in Figure 4. This makes the information much more readable—much like what has been done since the late sixties with Report Writers for mainframe LP programs such as MPS and Allegro. Again, the shaded rows indicate binding constraints or bounds.

Non-Negativity Conditions

If a variable is not used in the solution, that is zero, it is bounded by the non-negativity condition, and the marginal profit will appear in the top part of the report. But because we define the MV as Reduced Cost-Shadow Price (both can’t be non-zero at once), then we will always compute the correct marginal value. To illustrate this

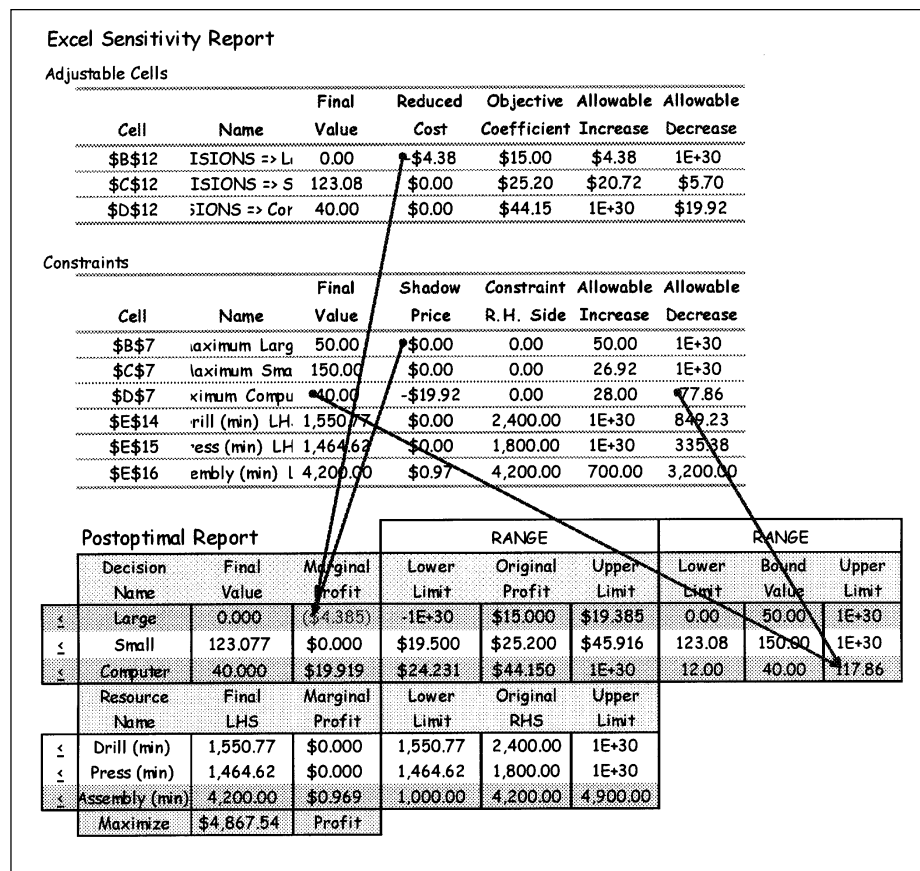


Figure 5: Marginal value for binding non-negativity constraint.

case, let us change the unit profit on Small Tables to \$15.00. The Excel Sensitivity Report and resulting Postoptimal Report are shown in Figure 5. For each Small Table we are forced to produce, we lose \$4.38 from 0 to infinity (1E+30).

Conclusion

By writing bounds constraints “backwards” and carefully constructing the bounds limits, we are able to reduce the size of an LP spreadsheet model and yet retain all the valuable information from a bounded variable simplex code. By reconfiguring the Excel Sensitivity Report, we can also construct a Postoptimal Report that managers can read and interpret. Of course, only the Postoptimal Report is printed, and the Excel Sensitivity Report is not shown. As with all of these column articles, the Excel spreadsheet (MetalTables.xls) is available on the *Decision Line* Web site. ■

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