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All different Constraint and the Traveling Sales Problem Heuristics (Part I)

by Rick Hesse, Feature Editor, Pepperdine University



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Back in the days of rotary calculators and vacuum tube computers with punch cards and plug-board compilers (IBM 650), I took my first OR/MS course, and the one problem that really caught my attention was the Traveling Salesman problem (to be politically correct now, let's call it the Traveling Sales Problem). It was so easy to state: Minimize cost, time, or distance for a salesperson to visit "n" cities once and only once and return to the start. But the problem quickly becomes impossible to enumerate all the possible solutions and choose the best one, even on those "new" computers—and still stretches the limit of any computer power available these days. Mathematicians call this type of problem NP-complete, and there are $n!/2$ possible solutions for a symmetrical problem (distance from city A to B is the same as B to A). This number grows astronomically large and computing even millions of routes per second would require millions of years to enumerate all possible solutions for a 30-city problem. Optimal solutions for this problem are difficult to achieve on a spreadsheet for even moderate-sized problems (up to 14 cities which means $14^2 = 196$ variables, solving it as an assignment problem with constraints).

Heuristics

If the problem is geometrically proportional, we know that the optimal tour cannot intersect itself. Figure 1 shows that the top path must be longer than the bottom one, because each dotted line is the hypotenuse of a triangle, which must be longer than either leg. There-

fore AC is longer than CD for triangle ACD and BD is longer than AB for triangle ABD.

But just because a tour has no intersections, does not mean that it is the minimum length. And if the time or cost between stops is not geometrically proportional (and what airline fares are?), then even looking at the loop does not yield any useful information. The well-known Two-Opt procedure (Lin) uses a heuristic to remove all sets of two links and reconnect to determine if there is a smaller tour cost or length, and has proven an effective way to deal with large problems. But this too is difficult to perform on a spreadsheet.

Traveling Sales Spreadsheet

A 15-city problem, compliments of Steve Sacks of the University of Connecticut is shown in Figure 2, with a heuristic solution that gives 558 total miles (Sacks). Steve (Stephen.Sacks@UConn.edu) has a nice Visual Basic program which allows students to attempt to find the optimal solution to this geometrically accurate TSP (and also 6-, 10-, and 13-city versions), and then the user can see the results of the "nearest neighbor" and 2-opt procedure. The spreadsheet shown in Figure 2 uses the distance table in E22:T37 to compute the distances along the path in D4:D18, and the graph points are plotted using the distances normalized on a 10-point scale in C22:D37. Although this path does not intersect itself, it is not optimal. Each number in A4:A18 indicates the beginning city in each link in the corresponding column B, and the city in column C is the ending city of the

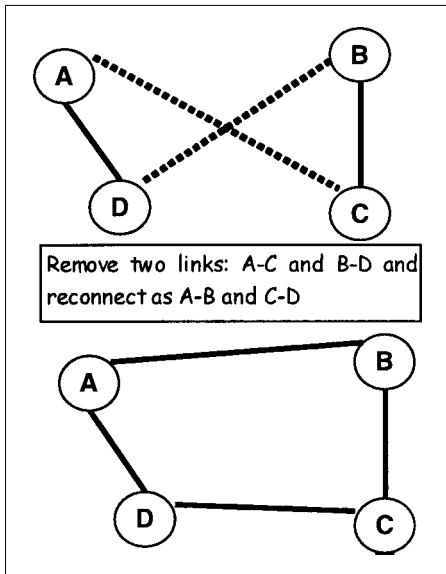


Figure 1: Triangle Proof of Non-intersecting Traveling Sales Tour.

link corresponding to the number in column A in the next row. This means 14→3 is Waterbury to Hartford for 23 miles. The tour “wraps around” with 11→14. The total tour length is in cell **D19** and is the sum of **D4:D18**. (The templates appearing in this column are available on the *Decision Line* website from links at www.decisionsciences.org/DecisionLine).

AllDifferent Constraint in Premium Educational Solver

The Premium Solver from Frontline Systems (Frontline Systems, <http://www.Solver.com>) and the Educational Premium Solver that accompanies most texts that teach management science has a feature that is extremely valuable for solving this problem. An integer constraint named AllDifferent is available which must be used with the Evolutionary Solver (also only available in the Premium Solver Platforms). This constraint allows the Solver to start from

an initial solution and then employs branch and bound to find the best possible ordering of the set of “n” numbers (1, 2, 3, ... n) that minimizes or maximizes the objective function. It does not perform implicit enumeration of all possibilities (so there is no guarantee of optimality) but does investigate a large number of possibilities from the current best solution. To set up and execute this heuristic, the “Premium” button must be selected in the Educational Premium Solver, so that the “Standard” button shows. Then you click on the drop-down box and select Standard Evolutionary for the type of solver. Then click the “Add” button and highlight the decision variables in **A4:A18** and select “dif” as shown in Figure 3.

Solution

The initial setup for the decision variables must be any number in the range 1→15 (in fact, can be all 1’s), and I have found that you don’t even need a good solution to start with. Figure 4 shows that just starting with the numbers in rank order gives an “etch-a-sketch” solution of 1,244 miles that is clearly not optimal.

After over 100,000 subproblems have been run, the Solver stops and states that it cannot improve the solution of 553 miles (shown in Figure 2). By running the Solver again, entering the same starting point of 1→15 (or choosing a new starting point of all 1’s or 15→1) the optimal solution of 548 miles is obtained, as shown in Figure 5. It may be that different computers will arrive at different answers due to round-off, etc. It is well worth trying several sets of starting points to see if an improvement can be made. Because this model does not have to be run as an assignment problem with break tours (to find the optimal), there are just “n” variables instead of “n²” variables, which means we can attempt a problem with 200 cities instead of just 14, since the standard Solver included in Excel (as well as the Educational Pre-

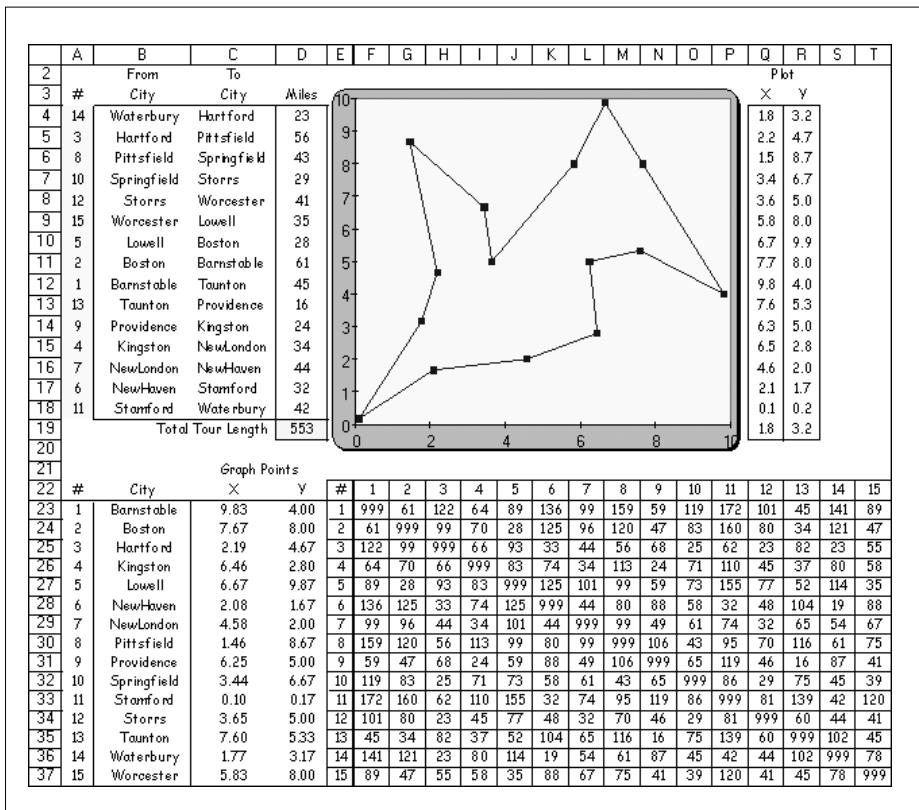


Figure 2: 15-city Problem Heuristic Solution.

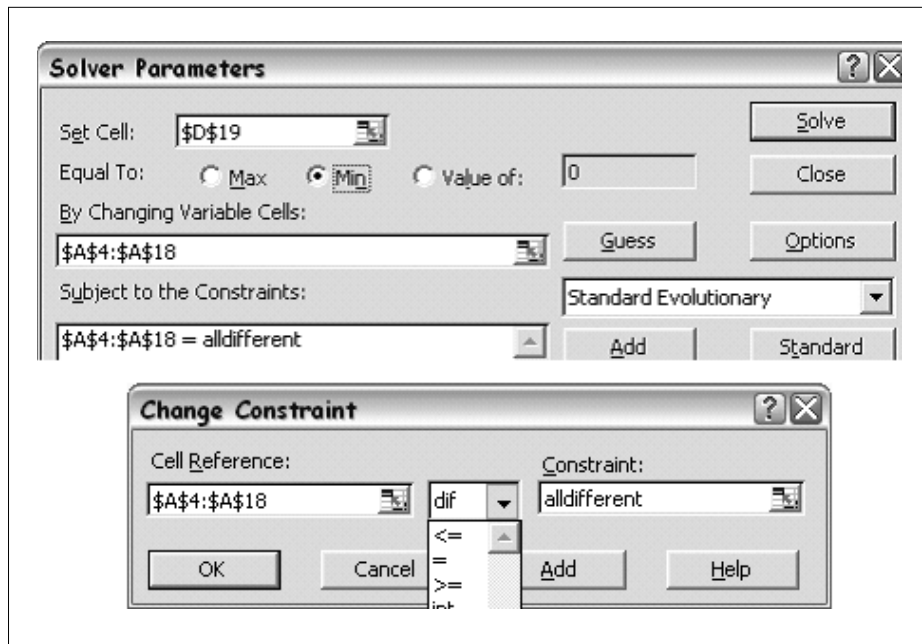


Figure 3: Evolutionary Solver Setup with alldifferent Constraint.

mium Solver) has a limit of 200 variables.

Traveling Sales String

With almost the same setup as the TSP, the Traveling Sales String (TSS) minimizes the distance, cost, or time to visit all the cities but not return to the start (which I call a string instead of a tour). Figure 6 shows the solution, and the only difference between the TSP and TSS template is that the string length is **SUM(D4:D17)** instead of adding the length back to the beginning. There are still 15 variables (alldifferent), and again, choosing different starting points may result in different solutions. But this is still a very efficient heuristic and allows up to 200 cities. Of course, any of the templates shown in this column can be expanded to work with larger problems and the distances between cities do not have to be geometrically proportional, nor does there have to be a graph of these points. As an illustrative exercise in class, it is nice to be able to “see” different solutions, but cer-

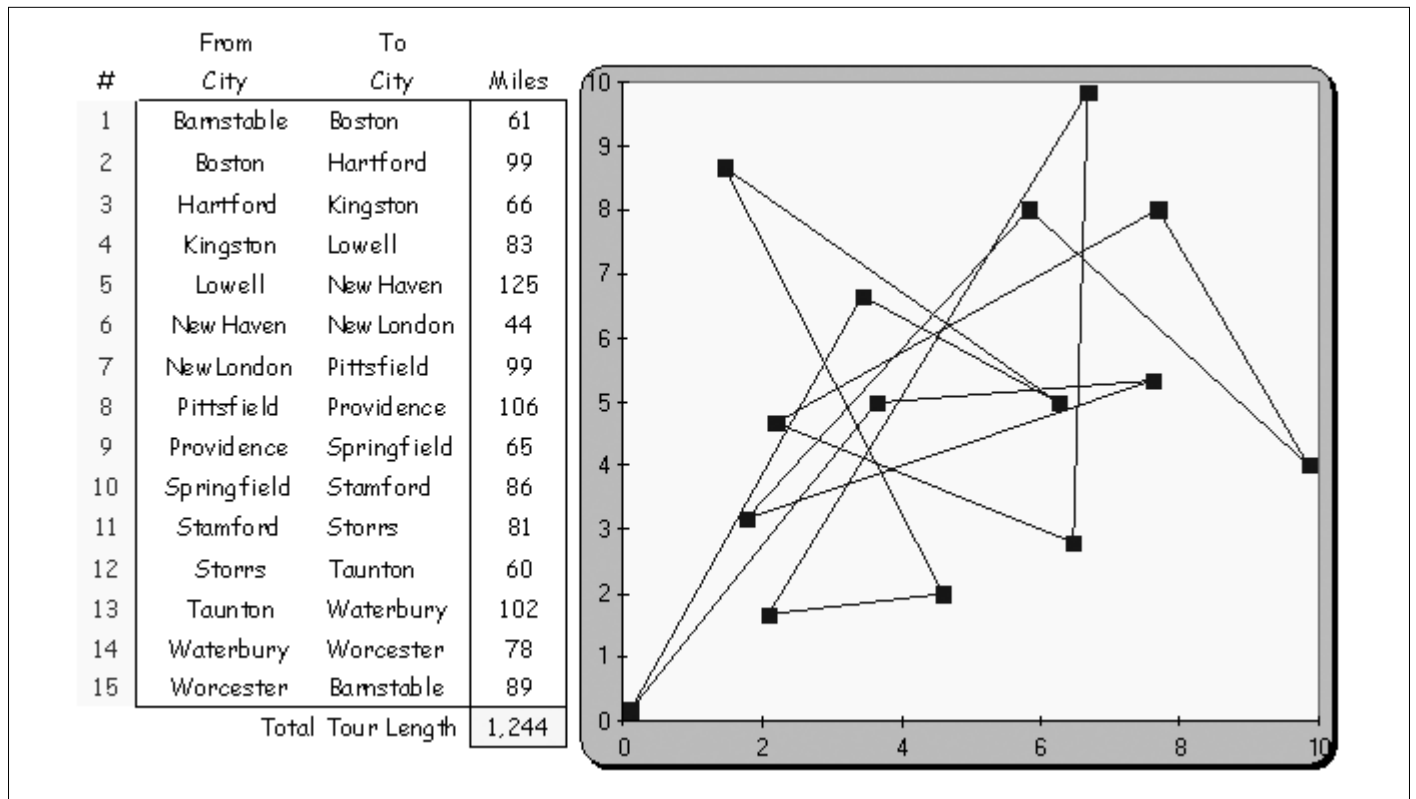


Figure 4: Initial Solution for 15-city Problem.

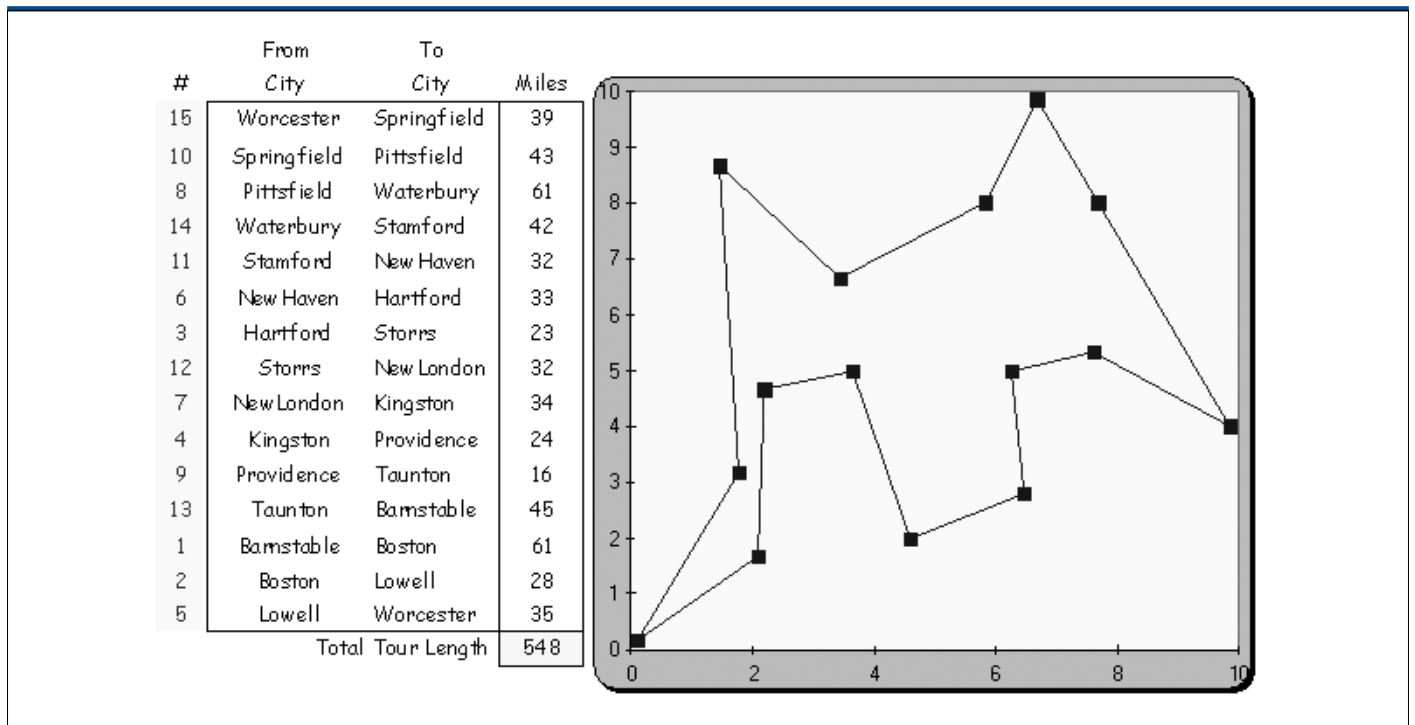


Figure 5: Optimal Solution Using the alldifferent Constraint.

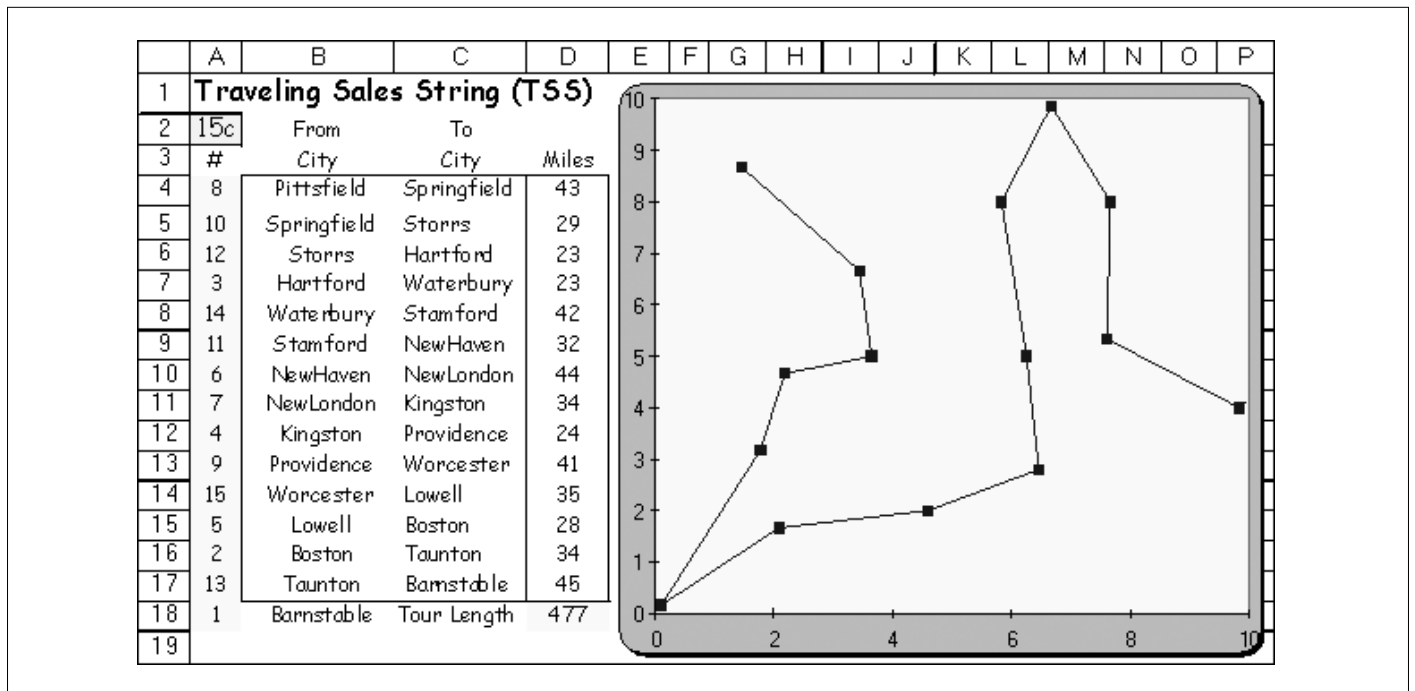


Figure 6: Traveling Sales String Solution.

tainly not necessary, especially for larger problems.

To Be Continued

In the next issue I will show how to make a change in the template to tackle

the Delivery Problem, where you can have more than one tour returning to the base and also attempt to keep the same number of stops for each tour.

References

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- Sacks, S. Department of Economics, University of Connecticut, Storrs, Conn. ■