■ RICK HESSE, Feature Editor, Pepperdine University

Alldifferent Constraint and the Delivery Problem Heuristic (Part II)

by Rick Hesse, Feature Editor, Pepperdine University

The Delivery Problem Tour (DPT) is a general case of the Traveling Sales Tour (TST) where the object is to minimize the time, distance or cost of visiting the Base (city) "n" times (n ≥ 1) and all other stops (cities) once and only once. "n" is the number of loops in the tour. Each loop could be traveled by a single bakery truck which returns to fill up the truck and go out again. Or the loops might be the number of bakery trucks needed to make all the deliveries simultaneously. This is a problem that has a large number of feasible solutions, and is not easy to solve optimally on a PC or without specialized software. That is where the heuristic shown below comes in handy.

Alldifferent Constraint

As with the Traveling Sales Tour and Traveling Sales String (Hesse, 2005) we can employ the Alldifferent constraint along with the Evolutionary Solver as a heuristic for the Delivery Problem. This is an option available only in the Educational Premium Solver, bundled with most texts in this field. This example involves visiting 15 cities in the Northeast with given mileage (Sacks, 1998). To make use of this Alldifferent constraint, we must augment the table of times by adding a duplicate row and column for each Base needed (in this case, two of them). Then the setup is the same as for the Traveling Sales Tour (TST). One solution is shown in Figure 1 and while it is a legitimate Delivery Problem solution with two loops, it is not practical, because one subtour visits only one city, Tauton, and the other has 12 stops. What is needed is a way to force a fairly equal number of stops between the return to the stop at the base.

Forcing Near Equal Stops Per Loop

A constraint must be added that uses the numbers in Column E to ensure that there are about half the stops (7) before returning to the Base. This is very tricky, because the two tours "wrap around," so the following formulas are used along with the solution shown in Figure 2, which is the optimal solution found using another technique (Hesse, 2005). A "2" is entered into **G2** to indicate that two loops are required.

F23: =TRUNC((MAX(E4:E19)-G2)/G2)
The minimum number of stops for each loop.

B22: **=VLOOKUP**(1,**$A$4:$E$19**,5,0)
The position of 1 – the home base.

B23: **=VLOOKUP**(2,**$A$4:$E$19**,5,0)
The position of 2 – also the home base.

C22: =SMALL($B$22:$B$23,A22)
Finds start position for first loop.

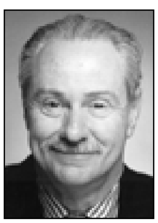
C23: =SMALL($B$22:$B$23,A23)
Finds start position for second loop.

D22: =C23-C22-1
Finds number of stops for first loop.

D23: =C22-C23-1+**E19**
Finds number of stops for second loop.

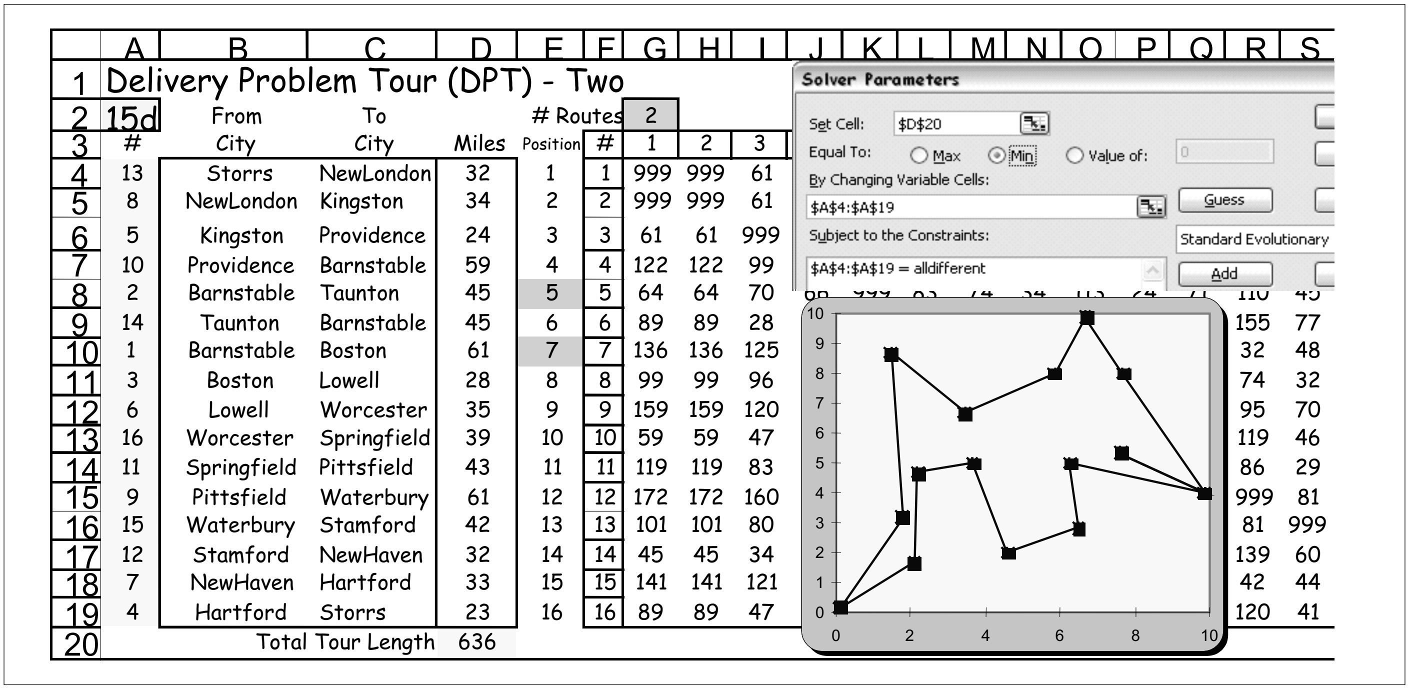
E23: =MIN(D22:D23)
The number of stops for the smallest loop.

This constraint attempts to divide the number of stops in half (for even



Rick Hesse *is professor of quantitative methods at Pepperdine University in the Graziadio Graduate School of Business and Management. He received his BS, MS, and DSc at Washington University School of Engineering in applied math and computer science. Dr. Hesse is the author of* Managerial Spreadsheet Modeling & Analysis *and* Applied Management Science: A Quick & Dirty Approach *(with Gene Woolsey), articles in numerous journals, and software for personal computers. Rick was the first professor to be awarded the Outstanding Civilian Service Medal by the Department of the Army at West Point in 1982, and was the winner of the Decision Sciences Institute's Innovative Instructional Award in 1981.*

Rick Hesse
Pepperdine University
Graziadio Graduate School of Business and Management
Malibu, CA 90265
rickhesse@aol.com



	A	B	C	D	E	F	G	H	I	R	S
1	Delivery Problem Tour (DPT) - Two										
2	15d	From	To		# Routes		2				
3	#	City	City	Miles	Position	#	1	2	3		
4	13	Storrs	NewLondon	32	1	1	999	999	61		
5	8	NewLondon	Kingston	34	2	2	999	999	61		
6	5	Kingston	Providence	24	3	3	61	61	999		
7	10	Providence	Barnstable	59	4	4	122	122	99		
8	2	Barnstable	Taunton	45	5	5	64	64	70		
9	14	Taunton	Barnstable	45	6	6	89	89	28	155	77
10	1	Barnstable	Boston	61	7	7	136	136	125	32	48
11	3	Boston	Lowell	28	8	8	99	99	96	74	32
12	6	Lowell	Worcester	35	9	9	159	159	120	95	70
13	16	Worcester	Springfield	39	10	10	59	59	47	119	46
14	11	Springfield	Pittsfield	43	11	11	119	119	83	86	29
15	9	Pittsfield	Waterbury	61	12	12	172	172	160	999	81
16	15	Waterbury	Stamford	42	13	13	101	101	80	81	999
17	12	Stamford	NewHaven	32	14	14	45	45	34	139	60
18	7	NewHaven	Hartford	33	15	15	141	141	121	42	44
19	4	Hartford	Storrs	23	16	16	89	89	47	120	41
20			Total Tour Length	636							

Figure 1: Original Solution for Delivery Problem with Alldifferent Constraint.

number of cities) and near half if the number of cities is odd. The constraint **$E$23 >= $F$23** is added to the Solver and then the Solver can be run. Starting with 1 → 2 → 3 → … → 16 yields the solution in Figure 2.

The tour length is now 731 miles, an increase of 203 miles over the Traveling Sales solution, and 95 more miles than the solution shown in Figure 1. The two loops of the tours take 368 and 363 miles, respectively. In this case, the two lengths are almost identical, but there is no guarantee that having the same number of stops will produce loops of near equal mileage. Of course different starting points may yield different solutions, so it pays to try different starting patterns with this heuristic. One could also just enter a "6" in **E23** to see what affect having at least 6 stops per loop might have. I got 727 miles with 6 stops (7 cities, 360 miles) and 8 stops (9 cities, 367 miles) starting from the solution in Figure 2. As with the Traveling Sales Tour, if the locations on the graph are geometrically accurate, the optimal loops will never have intersections within each loop.

Adding More Loops

For three loops, the template has been again augmented by another row and column with the same values for cities 1, 2, and 3 and now there are 17 "cities" in the problem. Another row of equations must also be added to ensure that each loop has close to an equal number of stops. There doesn't seem to be any easy way to augment the number of loops without changing the table or adding equations, but the pattern is not too difficult to establish. Figure 3 shows the results of starting at 1 → 2 →3… → 17 with a total distance of 910 miles. In this case, with 17 cities (the base is counted 3 times now), **F25** results in needing at least 4 stops for each loop to try to equalize the number of stops for each loop.

The result is one loop of 6 stops (451 miles!) and two loops of 4 stops (235 and 224 miles). Although this might be the minimum total miles, the large loop is much longer than the other and solution is certainly not balanced. Perhaps 3 loops with 5, 5, and 4 stops might work better, and a constraint added to make the maximum number of stops ≤ 5. When this is done, after running from the starting point in Figure 3 and then running again, I get a total of 916, with loops of 4 stops (209 miles), 5 (345) and 5 (362). This is some improvement in the disparity between the longest and shortest loop. Again, different starting points, or even running for the last stopping point, might yield solutions that are more acceptable. If four loops are allowed, I get 984 total miles, with loops of 5 stops (377 miles), 3 (231), 3, (213), and 3 (163) after running several times. This illustrates the problem of this branch and bound heuristic—it is dependent upon starting points, type of computer and other variables, but it is quick and easy to run many times. If the number of stops for each loop is constrained between 3 and 4, I am able to get a mileage of 976 miles, even less than when just forcing a minimum of 3 stops per loop.

Other Considerations

From the results of the models shown, it is obvious that maybe constraints that

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Delivery Problem Tour (DPT) - Two Tours																
2	15d2	From	To		# Routes	2											
3	#	City	City	Miles	Position	#	1					6	7	8	9	10	11
4	1	Barnstable	Taunton	45	1	1	999	999	61	122	64	89	136	99	159	59	119
5	14	Taunton	Providence	16	2	2	999	999	61	122	64	89	136	99	159	59	119
6	10	Providence	Springfield	65	3	3											
7	11	Springfield	Pittsfield	43	4	4											
8	9	Pittsfield	Worcester	75	5	5											
9	16	Worcester	Lowell	35	6	6											
10	6	Lowell	Boston	28	7	7											
11	3	Boston	Barnstable	61	8	8											
12	2	Barnstable	Kingston	64	9	9											
13	5	Kingston	NewLondon	34	10	10											
14	8	NewLondon	NewHaven	44	11	11											
15	7	NewHaven	Stamford	32	12	12											
16	12	Stamford	Waterbury	42	13	13											
17	15	Waterbury	Hartford	23	14	14											
18	4	Hartford	Storrs	23	15	15											
19	13	Storrs	Barnstable	101	16	16											
20			Total Tour Length	731													
21	#	Position	Reorder	#stops													
22	1	1	1	7	Min ≥ Stops												
23	2	9	9	7	7	7											

Subject to the Constraints:

$A$4:$A$19 = alldifferent

$E$23 >= $F$23

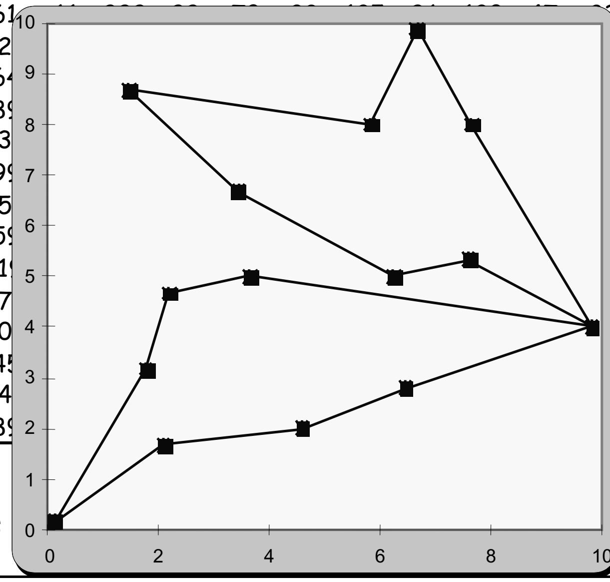


Figure 2: Constraint and Optimal Solution.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Delivery Problem Tour (DPT) - Three Tours																					
2	15e	From	To		# Routes	3																
3	#	City	City	Miles	Position	#	1	2	3	4												
4	7	Lowell	Boston	28	1	1	999	999	999	61												
5	4	Boston	Barnstable	61	2	2	999	999	999	61												
6	2	Barnstable	Springfield	119	3	3	999	999	999	61												
7	12	Springfield	Pittsfield	43	4	4	61	61	61	999												
8	10	Pittsfield	Hartford	56	5	5	122	122	122	99												
9	5	Hartford	Waterbury	23	6	6	64	64	64	70												
10	16	Waterbury	Stamford	42	7	7	89	89	89	28												
11	13	Stamford	NewHaven	32	8	8	136	136	136	125												
12	8	NewHaven	Barnstable	136	9	9	99	99	99	96												
13	3	Barnstable	Kingston	64	10	10	159	159	159	120												
14	6	Kingston	NewLondon	34	11	11	59	59	59	47												
15	9	NewLondon	Storrs	32	12	12	119	119	119	83												
16	14	Storrs	Taunton	60	13	13	172	172	172	160												
17	15	Taunton	Barnstable	45	14	14	101	101	101	80												
18	1	Barnstable	Providence	59	15	15	45	45	45	34												
19	11	Providence	Worcester	41	16	16	141	141	141	121												999
20	17	Worcester	Lowell	35	17	17	89	89	89	47	55	58	35	88	67	75	41	39	120	41	45	78
21			Total Tour Length	910																		
22	#	Position	Reorder	# stops																		
23	1	15	3	6																		
24	2	3	10	4	Min ≥ Stops																	
25	3	10	15	4	4	4																

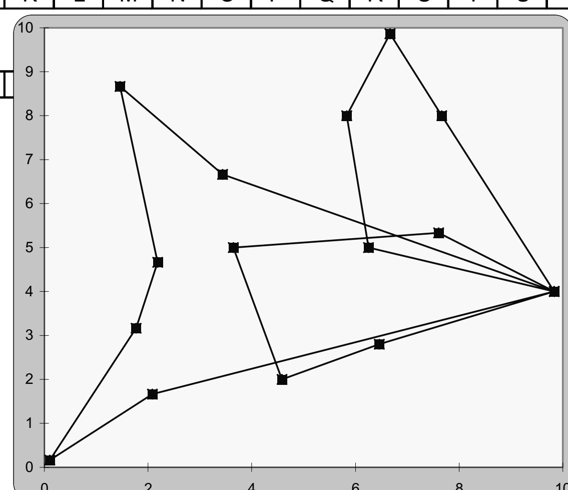


Figure 3: Delivery Problem with Three Loops.

try to equalize the mileage (or cost, time) for the loops might be better than the number of stops. Also different starting points might yield better solutions. As always, the workbook for the Excel templates is available on the *Decision Line* website, and will allow you and your students to play around with these examples or modify the template for other data.

References

Hesse, R. (2005). Alldifferent constraint and the traveling sales problem heuristics – Part I. *Decision Line*, *36*(4), July.

Sacks, S. (1998). Department of Economics, University of Connecticut, Stohrs, Conn. ■

Tips for Doctoral Students—*Getting the Most from the Annual Meeting*

The Decision Sciences Institute Annual Meeting provides an opportunity for doctoral students to network, develop professional skills, interview for faculty positions—and have a good time!

For students who want to know how to get the most out of the Annual Meeting, see "Tips for Doctoral Students" on the DSI Web site at

http://www.decisionsciences.org/doc_tips.htm

DSI Director of Development and Corporate Relations Vacancy Announced

Tim Smunt's term as Director of Development and Corporate Relations ends March 2006. Pursuant to Institute procedures, the Board of Directors is seeking qualified candidates for this position. The Director of Development and Corporate Relations serves a three-year term and may be reappointed for a second three-year term. Tim has indicated that due to other administrative responsibilities that he will not be able to serve a second term. Anyone interested in the position should contact Carol Latta at the address below.

The Director of Development and Corporate Relations oversees and coordinates major sponsorships with corporations and other business organizations. The Director is responsible for developing additional major sponsorships for the support of the Annual Meeting and for cultivating and maintaining long-term relationships with corporations. The Director personally presents an annual report to the Institute's Board during its January Meeting, as well as providing a written report to the Board. The Director of Development and Corporate Relations may be asked to attend other board meetings. Thus, the position requires significant travel support from the Director's institution.

The major responsibilities of the Director of Development and Corporate Relations include:

1. Develop and maintain a sponsorship framework for the annual meetings.
2. Solicit donations from publishers and other organizations.
3. Manage and expand contact e-mail lists of prospective sponsors from both universities and corporations.
4. Communicate with potential sponsors through email, letters and telephone.
5. Cultivate and maintain long-term relationships with corporations on behalf of the Institute.
6. Locate and contact volunteers to develop more and larger corporate sponsorships.
7. Coordinate with the Executive Director of DSI on all matters related to sponsorships.
8. Report to the Board of Directors on Institute on sponsorship activities and initiatives related to developing long-term corporate relationships with the Institute.

Questions about the position may be directed to the current Director of Development and Corporate Relations, Tim Smunt, Wake Forest University, at (336) 758-4423 or tim.smunt@mba.wfu.edu. All interested parties should submit the following to Carol Latta, Executive Director, Decision Sciences Institute, 35 Broad Street, Atlanta, GA 30303 by no later than January 15, 2006:

1. Curriculum vita
2. Statement of activities and service provided to the Institute
3. Statement of interest and availability to serve a three-year term
4. Statement of qualifications and experience related to the position
5. Description of institutional commitment for the support of the director's job functions for a three year period. ■