■ RICK HESSE, Feature Editor, Pepperdine University

PERT Simulation in Excel

by Rick Hesse, Feature Editor, Pepperdine University

COMAP (Consortium for Mathematical Applications Problems) and INFORMS have made up a simple PERT example of the time to turn around an airplane (offload passengers and baggage and then load the plane after cleaning the cabin). I use this simple example with probabilistic outcomes for activity times, and demonstrate how the critical path can vary by using the random number function in Excel and data tables.

Problem

The problem is shown in network form in Figure 1 with the mean and standard deviation (assuming normally distributed) shown on the activity.

There are only three possible paths, and they are A→B→D→E (55 minutes), A→C→E (47), and A→C→D→E (52). Activity C→D is a dummy activity, to indicate that regulations will not allow passengers to be boarded until the bags have been unloaded.

Excel Simulation

The **NORMSINV** (normal standard inverse) and **RAND** (uniform random number) functions are used to simulate the number of standard deviations from the mean for the five activities. In this case, Figure 2 shows the longest path is A→B→D→E with an expected time of 55.0 minutes but simulated time of 55.4.

G4: =**NORMSINV**(**RAND**()) and copied into **G5:G6** and **G8:G9**.
D4: =**MAX**(**E4**-2**F4**,**ROUND**(**E4**+**F4***G4**,1)) and copied into **D5:D6** and **D8:D9**.

The formulas in column D insure that no values are less than two standard deviations below the mean.

D10: =**D4**+**D6**+**D9**
D11: =**D5**+**D8**
D12: =**D5**+**D9** determine each path length.
F10: =(**D10**=**MAX**(**$D$10:$D$12**))***C10** and copied to **F11:F12** shows which path is longest.
F13: =**SUM**(**F10:F12**)

Let us ignore **G10:G12** for now.

ACTIVITIES	ACTIVITIES		Mean	Stdv
Unload Passengers	A	B	13.0	4.0
Unload Bags	A	C	25.0	8.0
Clean Cabin	B	D	15.0	4.0
Dummy Branch	C	D		
Load Bags	C	E	22.0	5.0
Load Passengers	D	E	27.0	8.0

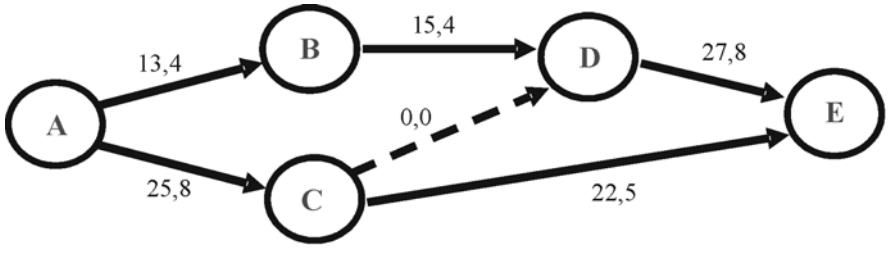

Figure 1. Event on activity diagram.

Rick Hesse
is professor of quantitative methods at Pepperdine University in the Graziadio Graduate School of Business and Management. He received his BS, MS, and DSc at Washington University School of Engineering in applied math and computer science. Dr. Hesse is the author of Managerial Spreadsheet Modeling & Analysis *and* Applied Management Science: A Quick & Dirty Approach *(with Gene Woolsey), articles in numerous journals, and software for personal computers. Rick was the first professor to be awarded the Outstanding Civilian Service Medal by the Department of the Army at West Point in 1982, and was the winner of the Decision Sciences Institute's Innovative Instructional Award in 1981.*

Rick Hesse
Pepperdine University
Graziadio Graduate School of Business and Management
Malibu, CA 90265
rickhesse@aol.com

Every time **<F9>** is pressed, the spreadsheet calculates another set of normal random numbers and conditional formatting highlights the longest (critical) path.

Replications

An easy way to replicate this simulation is to make up a One-way Data Table (row table) to the right to capture 50 runs. In Figure 3, the Data Table in **J3:BH11** fools Excel into running the simulation 50 times, with the row input cell being **J3** (or any useless cell on the spreadsheet). The values in **H4:H13** give information about the 50 replications, and in this case showthat Path #1 was critical 54% of the time, but that #2 was critical 28%, and #3 critical 18% of the time. The average length of time for a turnaround was 57.7 minutes (versus the longest expected 57 minutes); given the calculated standard deviation from the 50 runs (8.5), the expected probability of turning around an aircraft in 60 minutes is 60.8% while 28 of the 50 replications were 60 minutes or less. The longest time was 76.2 minutes and the shortest was 41.8 minutes. **G10:G12** uses the **COUNTIF** statement to determine the percentages for each path being critical, with A-B-D-E being the leader with 54%. The most critical activity (see **H4:H9**) is Loading Passengers at 72%, so the airline should concentrate on reducing that time.

	A	B	C	D	E	F	G
3	ACTIVITIES	ACTIVITIES		Minutes	Mean	Stdv	NRN
4	Unload Passengers	A	B	16.6	13.0	4.0	0.90
5	Unload Bags	A	C	24.1	25.0	8.0	-0.11
6	Clean Cabin	B	D	12.2	15.0	4.0	-0.71
7	Dummy Branch	C	D				
8	Load Bags	C	E	19.5	22.0	5.0	-0.49
9	Load Passengers	D	E	26.6	27.0	8.0	-0.05
10	Expected time	55	1	55.4	A-B-D-E	1	##
11		47	2	43.6	A-C-E		##
12		52	3	50.7	A-C-D-E		##
13					Path	# 1	

Figure 2. Simulation of airplane turnaround.

Each time **<F9>** is pressed, the runs 50 more runs in the data table and one more in **A3:G13**. It demonstrates the variability of not only each activity being critical, but each path, too. Conditional formatting is used in **A4:E8** and **D10:E12** to show which activities and path are critical for the single run.

Other Distibutions

You may wish to change the normal distribution to a Beta distribution or the Triangular distribution, which are not symmetrical, to illustrate how they may change the results. This is a bit more complicated, and you may want to use Crystal Ball or @Risk to accomplish this. As usual, the Excel template is available at the usual *Decision Line* website. ■

	A	B	C	D	E	F	G	H	I	J	K	L	BG	BH
1	AIRLINE TURNAROUND SIMULATION													
2	Airline													
3	ACTIVITIES	ACTIVITIES		Minutes	Mean	Stdv	NRN	Critical?		50 Trials	1	2	49	50
4	Unload Passengers	A	B	16.6	13.0	4.0	0.90	54%		1	1	1		1
5	Unload Bags	A	C	24.1	25.0	8.0	-0.11	46%					1	
6	Clean Cabin	B	D	12.2	15.0	4.0	-0.71	54%		1	1	1		1
7	Dummy Branch	C	D					18%					1	
8	Load Bags	C	E	19.5	22.0	5.0	-0.49	28%						
9	Load Passengers	D	E	26.6	27.0	8.0	-0.05	72%		1	1	1	1	1
10	Expected time	55	1	55.4	A-B-D-E	1	54.0%	57.7	Minutes	55.4	48.2	48.9	63.4	47.7
11		47	2	43.6	A-C-E		28.0%	8.5	Stdev	1	1	1	3	1
12		52	3	50.7	A-C-D-E		18.0%	60.8%	E[p(<=60)]	76.2	Max			
13					Path	# 1		56.0%	O[p(<=60)]	41.8	Min			
14								60						

Figure 3. Replications of PERT simulation.