CHANCE CONSTRAINED PROGRAMMING FOR SECURITY ANALYSIS AND PORTFOLIO MANAGEMENT

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ABSTRACT

The focus of this research paper is to develop a model for security analysis and investment decisions using multiobjective chance constrained programming (CCP). Using relevant variables and constraints a deterministic nonlinear programming model is derived using CCP technique. The model's efficiency and effectiveness is also evaluated as applied to a sample of stocks selected from the Dow Jones Industrial Average compiled into a high yielding portfolio.

Keywords: Portfolio, Random Variable, Chance Constraint, Multiobjective Programming.

INTRODUCTION

In the world of investment, investors want to earn the highest expected return from the portfolio. The rate of expected return depends on the level of risk tolerance. The expected return from a portfolio of stocks is a combination of dividend and price yields. As a consequence, portfolio selection and security analysis always becomes a vital area for decision making. Markowitz [1], a Nobel laureate, devised modern portfolio theory in 1952. Since then, portfolio theories and models have emphasized the importance of portfolios, risk, the correlations between securities, and diversification. Sharpe [2, 3, 4] and Stone [5] applied linear programming (LP) to construct optimal efficient portfolios and demonstrated how acceptable results can be achieved while avoiding the limitations of mean-variance models. Recently, Chance Constrained Programming (CCP) models have been widely used for providing optimized solutions to problems that have multiple and conflicting objectives and being kwon as multi objective chance constrained programming models [6].

In this paper, we present an efficient portfolio model for security analysis and investment decisions using multiobjective chance constrained programming (MOCCP). Different parameters in a portfolio model are considered as random variables for capturing uncertainty. Therefore, some of the constraints are expressed as chance constraints. Assuming random parameters of the model as normal random variables, we derive the equivalent deterministic nonlinear programming model.

MOCCP PROBLEM AND ITS DETERMINISTIC EQUIVALENT FORM

A CCP problem involving more than one objective function is known as a multiobjective chance constraint programming (MOCCP) problem. It can be stated as

$$\max : \sum_{j=1}^{n} c_{kj} x_j , k = 1,2,..., K_1 \tag{1}$$

$$\min: \sum_{j=1}^{n} c_{kj} x_j, k = K_1 + 1, K_1 + 2, ..., K \quad (2)$$

subject to
$$\Pr(\sum_{j=1}^{n} a_{ij} x_j \le b_i) \ge 1 - \alpha_i, \quad i = 1,2,...,m \quad (3)$$

$$x_j \ge 0, \; j = 1,2,...,n, \quad \alpha_i \in (0,1), \; i = 1,2,...,m \quad (4)$$

where x_j is the j-th ($j = 1,2,...,n$) decision variable, c_{kj}, a_{ij} and b_i are random variables, α_i is the specified level of significance of the i-th $(i = 1,2,...,m)$ probabilistic constraint. Objective functions as well as some constraints of the model (1)-(4) involve random variables. Next, we derive the deterministic equivalent of the objective functions and constraints.

The determinant nonlinear programming form of the model (1)-(4) can be obtained as:

$$\min: \sum_{k=1}^{K_1} \lambda_k + \sum_{k=K_1+1}^{K} \lambda'_k \quad (5)$$

subject to
$$E(z_k - \sum_{j=1}^{n} c_{kj} x_j) + \phi^{-1}(1 - \beta_k) \sqrt{\mathrm{var}(z_k - \sum_{j=1}^{n} c_{kj} x_j)} \le \lambda_k, \; k = 1,2,...,K_1 \quad (6)$$

$$E(\sum_{j=1}^{n} c_{kj} x_j - z'_k) + \phi^{-1}(1 - \beta'_k) \sqrt{\mathrm{var}(\sum_{j=1}^{n} c_{kj} x_j - z'_k)} \le \lambda'_k, \; k = K_1 + 1, K_1 + 2, ..., K \quad (7)$$

$$E(\sum_{j=1}^{n} a_{ij} x_j - b_i) + \phi^{-1}(1 - \alpha_i) \sqrt{\mathrm{var}(\sum_{j=1}^{n} a_{ij} x_j - b_i)} \le 0, \; i = 1,2,...,m, \quad (8)$$

$$x_j \ge 0, \; j = 1,2,...,n, \; \alpha_i \in (0,1), \; i = 1,2,...,m, \; \lambda_k, \lambda'_k \ge 0 \quad (9)$$

where, $z_k = \max: \sum_{j=1}^{n} c_{kj} x_j, k = 1,2,...,K_1$ and $z'_k = \min: \sum_{j=1}^{n} c_{kj} x_j, k = K_1 + 1, K_1 + 2, ..., K$.

PORTFOLIO MODEL DEVELOPMENT

To formulate the MOCPP model for stock-based portfolio, the following notations are defined first: s = index for the proportion of money invested in stock $s \in \{1,2,\cdots,S\}$; X_s = proportion of money invested in stock s; R_s = annual rate of return earned from the stock *s*, a random variable; D_s = dividend or interest yield on stock *s*, a random variable; β_s = measure of risk associated with stock *s*, a random variable; U_s = amount of money invested on stock *s*, a random variable; Q_s = quality index of stock *s;* $Q_{\min}$ = minimum acceptable quality index, a random variable; PE_s = price earning ratio (company's financial performance); $PE_{\max}$ = maximum company's financial performance, a random variable; $R_{\max}$ = maximum acceptable annual rate of return; $D_{\max}$ = maximum annual income from dividend; $\beta_{\min}$ = risk of stock; a, a', a'' = level of significance of portfolio price earning ratio, analyst's rating, and investment diversification constraints, respectively.

Objective functions of the model

(i) **Portfolio's risk:** The portfolio's beta is called systematic risk and is measured as the sensitivity of a stock's market returns. This objective is expressed as: $\min : \sum_{s=1}^{S} \beta_s X_s$.

(ii) **Annual dividend:** In terms of annual dividend income, the objective is to earn the highest yield from all stocks. This objective is expressed as: $\max : \sum_{s=1}^{S} D_s X_s$.

(iii) **Annual return:** In terms of annual return, the objective is to earn the highest return from all stocks. This objective is expressed as: $\max : \sum_{s=1}^{S} R_s X_s$.

Constraints of the model

(i) **Portfolio's price earning ratio**: The price earning ratio of each company can be obtained from past data and the average of this could be used as a future value. It can be expressed as:

$$\Pr(\sum_{s=1}^{S} PE_s X_s \le PE_{\max}) \ge 1 - a \qquad (10)$$

(ii) **Analyst's rating:** The quality rating of the individual company is a subjective assessment. It is expressed as:

$$\Pr(\sum_{s=1}^{S} Q_s X_s \ge Q_{\min}) \ge 1 - a' \qquad (11)$$

(iii) **Investment:** The decision maker must utilize total funds available. So, $\sum_{s=1}^{S} X_s = 1$ (12)

(iv) **Investment diversification:** To minimize risk, an investor should not invest more than a certain proportion of his money in any single stock. This proportion may be random in nature. Therefore, the investment diversification constraint may be constructed as a chance constraint as follows:

$$\Pr(X_s \le U_s) \ge 1 - a'' , \quad s = 1, 2, \cdots, S \qquad (13)$$

APPLICATION

We considered a sample of 25 stocks from the Dow Jones Industrial Average to demonstrate the efficiency of the model. The relevant financial data for the problem is given in Table 1. The application of the model is based on the assumptions – (i) the holding period of the portfolio is assumed to be one year. If the holding period is changed, the goals, objectives and constrained may need to be redefined, (ii) the expected rate of return on each stock is assumed to be at least a fair, accurate and reliable return to make predictions for the next year, (iii) the returns on investments don't consider tax factor and (iv) the stocks selected can be purchased or sold in the market at any time which means that these securities are highly liquid securities.

Table 1: Required financial data

	Name	$E(R_s)$	$\sigma^2(R_s)$	PE_s	$E(D_s)$	$\sigma^2(D_s)$	$E(\beta_s)$	$\sigma^2(\beta_s)$	Q_s
1	Alcoa Inc	2.57	0.60	13.27	0.68	0.08	1.43	0.30	5
2	Amer Intl Group	5.37	0.98	12.49	0.66	0.08	1.10	0.12	5
3	Amer Express Inc	2.99	0.63	18.63	0.60	0.07	0.94	0.10	5
4	Boeing Co	2.81	0.52	31.56	1.40	0.19	0.69	0.08	4
5	Citigroup Inc	4.31	0.59	11.85	2.16	0.41	0.57	0.08	5
6	Caterpillar Inc	5.17	0.88	12.90	1.20	0.27	1.88	0.22	5
7	Du Pont	3.38	0.73	14.52	1.48	0.30	1.14	0.18	5
8	Walt Disney C	2.06	0.35	16.70	0.31	0.02	1.07	0.16	5
9	Gen Electric Co	2.00	0.29	17.61	1.12	0.26	0.49	0.06	4
10	Home Depot Inc	2.79	0.32	13.17	0.90	0.09	1.28	0.18	5
11	Honeywell Intl Inc	2.52	0.59	18.31	1.00	0.09	0.95	0.10	5
12	Hewlett Packard	2.31	0.41	17.48	0.32	0.05	1.77	0.28	5
13	Intl Business Mach	6.11	1.01	15.58	1.20	0.39	1.82	0.32	5
14	Intel	0.86	0.08	22.30	0.45	0.07	2.03	0.70	4
15	Johnson & Johnson	3.73	0.49	16.10	1.50	0.24	1.35	0.44	5
16	JP Morgan Chase	4.04	0.74	11.94	1.36	0.27	0.69	0.09	5
17	Coca Cola Co	2.16	0.26	22.44	1.36	0.27	0.61	0.08	4
18	McDonalds	2.83	0.36	15.83	1.00	0.10	1.35	0.13	4
19	3M Company	5.06	0.71	15.08	1.92	0.22	0.69	0.05	5
20	Altria Group Inc	5.71	0.96	11.95	3.44	0.43	0.78	0.09	4
21	Merck Co Inc	2.03	0.24	22.21	1.52	0.19	0.78	0.09	4
22	Microsoft	1.17	0.17	23.71	0.40	0.04	0.88	0.11	4
23	AT&T Inc	1.89	0.20	20.91	1.42	0.21	0.48	0.06	4
24	Verizon	2.12	0.31	17.84	1.62	0.38	0.66	0.04	5
25	Wal-Mart Stores	2.71	0.15	17.51	0.88	0.08	0.09	0.02	5

RESULTS

Using table 1, we solved the following two sub-problems to calculate the maximum acceptable annual return and annual income from dividend. We set the risk of stock as the expected risk of all stocks.

$$R_{\max} = \max : \sum_{s=1}^{S} \overline{R}_s X_s$$

subject to (3.1)-(3.4) and (2.4), $\overline{R}_s = E(R_s)$

$$D_{\max} = \max : \sum_{s=1}^{S} \overline{D}_s X_s$$

subject to (3.1)-(3.4) and (2.4), $\overline{D}_s = E(D_s)$, $\beta_{\min} = E(\beta_s)$

In solving the model, we needed to calculate the values of $\beta_{\min}$, $R_{\max}$ and $D_{\max}$. These were obtained as 1.02, 5.05 and 1.92. Confidence level of risk, annual return and dividend was set as 98.93%. We also set, $PE_{\max} \sim N(20.28, 3.02^2)$, $Q_{\min} \sim N(4.0, 0.04^2)$, $U_s \sim N(0.15, 0.008^2)$, and $a = a' = a'' = 0.05$.

Table 2: Results

	Name	Allocation (%)	$E(R_s)$	$E(D_s)$	$E(\beta_s)$
1	Alcoa Inc	0.00%	2.57	0.68	1.43
2	Amer Intl Group	13.69%	5.37	0.66	1.1
3	Amer Express Inc	1.47%	2.99	0.6	0.94
4	Boeing Co	0.00%	2.81	1.4	0.69
5	Citigroup Inc	13.69%	4.31	2.16	0.57
6	Caterpillar Inc	13.69%	5.17	1.2	1.88
7	Du Pont	4.28%	3.38	1.48	1.14
8	Walt Disney C	0.00%	2.06	0.31	1.07
9	Gen Electric Co	0.00%	2	1.12	0.49
10	Home Depot Inc	0.00%	2.79	0.9	1.28
11	Honeywell Intl Inc	0.00%	2.52	1	0.95
12	Hewlett Packard	0.00%	2.31	0.32	1.77
13	Intl Business Mach	13.69%	6.11	1.2	1.82
14	Intel	0.00%	0.86	0.45	2.03
15	Johnson & Johnson	0.00%	3.73	1.5	1.35
16	JP Morgan Chase	13.69%	4.04	1.36	0.69
17	Coca Cola Co	0.00%	2.16	1.36	0.61
18	McDonalds	0.00%	2.83	1	1.35
19	3M Company	12.13%	5.06	1.92	0.69
20	Altria Group Inc	13.69%	5.71	3.44	0.78
21	Merck Co Inc	0.00%	2.03	1.52	0.78
22	Microsoft	0.00%	1.17	0.4	0.88
23	AT&T Inc	0.00%	1.89	1.42	0.48
24	Verizon	0.00%	2.12	1.62	0.66
25	Wal-Mart Stores	0.00%	2.71	0.88	0.09
Weighted Average			5.01	1.68	1.08

The MOCCP model for the portfolio problem was formulated using the given data and executed using LINGO 10.0. The obtained result is presented in Table 2. It shows that the investor can achieve an average return of 5.01 percent including 1.68 percent dividend yield. If the investor had constructed the portfolio of nine companies (#2, 3, 5, 6, 7, 13, 16, 19, 20) with equal weight without using the model, he would have expected the return of 4.68 percent and dividend yield of 1.56 percent. Although, the value of beta has only gone up from average of 1.07 to 1.08, the variance of returns has gone down significantly from 1.01 percent to 0.62 percent.

CONCLUSION

The model developed in this study focuses on a new chance constrained programming formulation to construct an efficient portfolio that incorporates multiple goals. The deterministic portfolio model is derived assuming the normality of the random parameters of the model. The model is flexible enough to be applied to other similar real world investment decision making problems.

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