# FORECASTING PRODUCT RETURNS FOR PRODUCTION PLANNING AND CONTROL IN REMANUFACTURING

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## ABSTRACT

There is a growing focus on sustainability in corporate objectives. Sustainable operations holds the key to increased profitability and effective management of environmental issues. Among the business processes used for sustainability is remanufacturing. Legislative trends, as evidenced in the European Union, are forcing manufacturers to take back and put products back into the market, after remanufacturing. In this study, we consider a manufacturer that also acts as a remanufacturer, and develop a generalized forecasting approach to determine the returns of used products, as well as integrate it with an inventory model to enable production planning and control.
**Keywords:** Sustainable operations, remanufacturing, forecasting, inventory control

## INTRODUCTION

There has been tremendous corporate interest in sustainability in the last two decades. Most major corporations in the Fortune 500 are now expanding their strategic goals to include sustainability issues (www.wal-mart.com, www.usa.canon.com/green, www.toyota.com, www.ge.com, www.dell.com/sustainabilityreport are a few of the examples). The definition of sustainable development as "meets the needs of the present without compromising the ability of future generations to meet their own needs" by the World Commission on Environment and Development has been discussed by [7] through the notion of economic and social costs. The role of operations in this regard is noted. This leads to the concept of sustainable operations. Sustainable operations management is the set of skills and concepts that allows a company to structure and manage its business process to be profitable, without sacrificing the needs of its employees and external stakeholders (e.g. shareholders, lenders etc…), whilst having a regard for the impact of its operations on people and the environment [5]. Due to increasing legislation and the realization that being 'green' can be profitable, sustainable operations has become an important issue in the field of operations. One set of the business processes used for sustainability is reuse activities. Among reuse activities, which include repair, remanufacturing, and recycling, there is an increased focus on remanufacturing [6].

Remanufacturing is the process by which products are recovered, processed and sold as like-new products in the same or separate markets. The EPA cites remanufacturing as an integral foundation of reuse activities and reports that less energy is used and fewer wastes are produced with these types of activities [2]. However, few guidelines are available to the practicing manager to aid in planning and controlling remanufacturing operations [4]. Production planning and control activities are more complex and difficult for remanufacturers partly due to the uncertainty in the timing of returns. The returns that are used for remanufacturing are products that are sold to the customer and are returned when their useful life is over or when the customer wants to trade in the product for an upgrade or another unit of the product. Toner cartridges and tires are examples of the former and computers, copiers, engines and compressors are examples of the latter. Predicting the volume of such returns is important for procurement decisions, production planning, and inventory and disposal management.

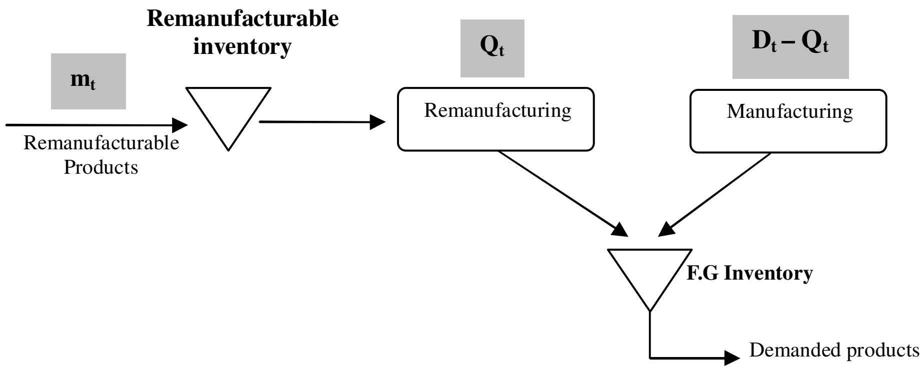
The environment considered in this study is product returns due to end-of-life (e.g. toner cartridges and cans). Forecasting of returns in this type of environment has been looked at by [9] for disposable cameras, and [3] for returnable bottles. In these environments, typically, only the sales and return volume in each period is known and not the timing of returns and sales. One method for forecasting returns in such an environment is to use a time series consisting of past return volumes to forecast future return volumes; however such a method would ignore past sales data. The key to forecasting returns is the observation that returns in any one period are generated by sales in the preceding periods. A transfer function model relating returns to previous sales is proposed in [3]. This model estimates the probability of a return after a certain number of periods, for a fixed set of data. In practice, the data is not fixed but is rather, augmented in each period as new sales and return information becomes available. This makes Bayesian estimation a better choice. A model which is conducive to data augmentation is the distributed lag model considered by [9]. Another advantage of using this type of model is that it generally involves the estimation of less parameters than the transfer method, thus less data is required for the analysis. In this study a Bayesian estimation approach is used to forecast returns, which is described in detail in a later section.

Once an appropriate forecasting mechanism has been developed for the returns, a second concern, from a production control and inventory management perspective is how to best use this information to minimize the production and inventory costs. In this study, the use of the newsvendor model is proposed to determine the amount of remanufactured product to make, that will minimize holding and production costs given the distribution of forecasted returns. Since the level of returns are forecasted periodically, the remanufacturing decision is also made periodically, however the decision of how much to remanufacture is only made for a single period at a time, and thus a single-period model such as the newsvendor model is appropriate for this type of situation.

## THE INVENTORY MODEL

Figure1 below shows the setup of the environment being considered. The aim of the model is to determine the amount to remanufacture ($Q_t$) in each period ($t$), based on the estimation of the returns distribution ($F_t(m)$) for each period. In the above setup, there are no disposals. This is valid in environments where 100% of the returns are remanufactured. Once the quantity to remanufacture is determined, the amount of new product to manufacture will be the difference between the amount to be remanufactured and the forecasted demand ($D_t$). Remanufacturing occurs during the period until ($Q_t$) units have been made. If there are not enough returns in inventory to make ($Q_t$) units by the end of the period, the remaining remanufactured quantity needed is satisfied via the manufacture of new products.

**FIGURE 1: A manufacturing-remanufacturing operation with stocking points**




The decision to manufacture new products, to cover the remaining quantity of remanufactured products required, is one made at some point during the period so as to have enough time to fulfill the quantity requirements of the period with a newly manufactured product. Overestimation of $Q_t$ (i.e. less returns arrive in period t, than predicted by $Q_t$) leads to overage costs, $c_o$. This cost can be interpreted as the differential cost of making a new product instead of a remanufactured product plus the cost of having to expedite or change the manufacturing schedule. Underestimation of $Q_t$ (i.e. more returns arrive in period t, than predicted by $Q_t$), leads to underage costs, $c_u$. This cost can be interpreted as the holding costs incurred at the remanufacturable inventory ($I_t$ ) plus the cost of changing the production schedule to allow for more remanufacturing to occur (or the opportunity cost of not satisfying demand by remanufacturing the "surplus" returns). Since there is remanufacturable inventory, the policy governing the actual volume of new returns, i.e., returns brought into inventory during period t, that will be remanufactured during period t is as follows: Remanufacture $Q_t$ - $I_t$ of new returns if $Q_t < I_t$; Do not remanufacture new returns if $Q_t \geq I_t$.

It is assumed that the remanufacturable inventory is initially zero (i.e. $I_0 = 0$). For each period's decision, the newsvendor model is used to find $Q_t$ according to the following formulation:

$$\min_{Q_t}\left\{C(Q_t) = c_0\int_0^{Q_t}(Q_t - m_t)dF_t(m_t) + c_u\int_{Q_t}^{\infty}(m_t - Q_t)dF_t(m_t)\right\} \quad (1)$$

This leads to the well known result, for $Q^*_t$, the quantity that minimizes the above:

$$F_t(Q_t^*) = \frac{c_u}{c_o + c_u} \quad (2)$$

If the estimated cumulative distribution function (cdf) $F_t(.)$, turns out to be discrete, then $Q^*_t$ is found by evaluating the following:

$$\sum_{m=0}^{Q_t^*-1} F_t(m) < \frac{c_u}{c_u + c_o} < \sum_{m=0}^{Q_t^*} F_t(m) \quad (3)$$

Thus, the only additional information required, in order to determine $Q^*_t$, is the estimate of the cdf of returns. This can be obtained from the forecasting model described in the next section.

## THE MODEL FOR FORECASTING RETURNS

### Model Description

Let $n_t$ and $m_t$ denote the sales and returns of products in month $t$, respectively, and $\varepsilon_t \sim N(0, \sigma^2)$. The returns based on sales in previous time periods can be estimated using the following distributed lag model:

$$m_t = r_D(1)n_{t-1} + r_D(2)n_{t-2} + ... + r_D(t-1)n_1 + \varepsilon_t \text{; for t=2,…} \quad (4)$$

Usually, a specific form of the distribution involving one or two parameters is assumed for the delay function (i.e. $r_D(k)$), which serves the purpose of reducing the number of parameters to be estimated. Distributions that have been used are the geometric or negative binomial with parameter $q$ (e.g. as in [9]). Here, we use the exponential distribution with a parameter $\lambda$. One advantage of using an exponential delay function is that it is a continuous function which allows for fractional periods to be estimated. Fractional periods can occur if the periods used to record returns and sale volumes are of unequal length. Another advantage of using an exponential delay function is that it is more consistent with the assumption of exponential inter-arrival times of returns used in inventory queuing models.

Using an exponential distribution, Equation 4 becomes:

$$m_t = \lambda e^{-\lambda} n_{t-1} + \lambda e^{-2\lambda} n_{t-2} + \lambda e^{-3\lambda} n_{t-3} ... + \lambda e^{-(t-1)\lambda} n_1 + \varepsilon_t \text{; for t=2, 3,…} \quad (5)$$

Using the Koyck transformation (i.e. subtracting $e^{-\lambda} m_{t-1}$ from both sides of equation 5) we obtain:

$$m_t = e^{-\lambda} m_{t-1} + \lambda e^{-\lambda} n_{t-1} + \varepsilon_t - e^{-\lambda} \varepsilon_{t-1} \text{; for t=2,3,…} \quad (6)$$

Setting $u_t = \varepsilon_t - e^{-\lambda}\varepsilon_{t-1}$ and $\mathbf{u} = (u_2, u_{3,....,} u_T)$, for a given set of (T) time periods, the covariance matrix for $\mathbf{u}$ is given by:

$$\Sigma_u = \sigma^2 G \text{ where } G_{ij} = \begin{cases} 1+\lambda^2, i = j \\ -\lambda, \ (i = j+1) \ \ or \ \ (j = i+1) \\ 0, otherwise \end{cases} \quad (7)$$

$i, j = 1,..., T-1$

Given the set of returns ($\mathbf{m} = (m_2, m_3, ......, m_T)$) and sales ($\mathbf{n} = (n_1, n_2, ......, n_{T-1})$) for periods 1,…,T the likelihood for the parameters is thus given by:

$$\ell(\lambda,\sigma^2) \propto \frac{|G|^{-T/2}}{\left(\sigma^2\right)^{T/2}} \exp\left[-\frac{1}{2\sigma^2}\left(\mathbf{m}-e^{-\lambda}\mathbf{m}_{-1}-\lambda e^{-\lambda}\mathbf{n}\right)' G^{-1}\left(\mathbf{m}-e^{-\lambda}\mathbf{m}_{-1}-\lambda e^{-\lambda}\mathbf{n}\right)\right] \quad (8)$$

Where $\mathbf{m}_{-1} = (m_1, m_2, ......, m_{T-1})$

The following conjugate priors are used for λ and $\sigma^2$: $\lambda \sim Gamma[\alpha_0, \beta_0]$ ; $\sigma^2 \sim I.G(\frac{v_0}{2}, \frac{v_0 s_0^2}{2})$

$s_0^2$ = sums of squares of returns and $v_0$, are prior parameters to be specified, details of how to specify $v_0$ are shown in [8]. Note that the inverted gamma distribution (I.G) can also be represented as the inverse of a scaled chi-squared random variable with appropriate degrees of freedom [8]. Thus an alternative representation of the conjugate distribution of the variance parameter is: $\sigma^2 \sim \frac{v_0 s_0^2}{\chi^2_{v_0}}$.

In Bayesian analysis, the prior distribution represents the beliefs of the decision maker about the unknown parameters expressed in a probabilistic statement. Since the performance of successive updates depends on the prior, its choice is important [8]. The problem of improper priors is avoided by using the above conjugate priors. A conjugate prior, for a parameter, is a distribution, for which the posterior is also of the same family. The inverse gamma distribution is the natural conjugate prior for $\sigma^2$ [8, p.24). The gamma distribution is the conjugate prior of $\lambda$ [8].

**Model Estimation**

The posterior distributions for $\lambda$ and $\sigma^2$ are estimated by making use of the random walk Metropolis-Hastings (M-H) algorithm to simulate draws from the posterior distribution of model parameters [1][8, p.86]. The details of the estimation procedure are provided below:

i) Start with initial values of $\lambda$ and $\sigma^2$

ii) Generate: $\lambda^{new} = \lambda^{old} + \varepsilon$; $\varepsilon \sim N(0, step^2)$; Step is a numerical value chosen to enable the algorithm to have sufficiently navigated the space where the posterior has high mass.

iii) Compute $\alpha = min\left\{1, \frac{\ell(\lambda^{new}, \sigma^2)\pi(\lambda^{new})}{\ell(\lambda^{old}, \sigma^2)\pi(\lambda^{old})}\right\}$; where $\pi(.)$ is the prior for λ

iv) With probability α, λ=$\lambda^{new}$, else λ=$\lambda^{old}$

v) Generate G using λ

vi) Generate: $\sigma^2_{new}|\boldsymbol{m}, \boldsymbol{n}, \lambda \sim \frac{v_1 s_1^2}{\chi^2_{v_1}}$; with $v_1 = v_0 + (T-1), s_1^2 = \frac{v_0 s_0^2 + (T-1)s^2}{v_0 + (T-1)}$

vii) Repeat ii-vi) as necessary

The above yields the following posterior distributions: $f(\sigma^2 | \mathbf{m}, \lambda)$ and $f(\lambda | \mathbf{m})$.
The posterior distribution for $m_t$ can be obtained by running a MCMC (monte carlo markov chain) Gibbs sampler for the specified likelihood shown in equation 9 below:

$$f(\sigma^2 \mid \mathbf{m}, \lambda)\ f(\lambda \mid \mathbf{m})\ \ell(\lambda, \sigma^2) \quad (9)$$

The cdf (it will be a discrete estimation) is then obtained by ordering the draws from lowest to highest. The main task will be customizing the Gibbs sampler code to run the appropriate MCMC to estimate the above. Such codes are prevalent in the literature on applied Bayesian methods (see [8]).

## CONCLUSION

To verify that the M-H algorithm could correctly estimate the distribution of the rate parameter ($\lambda$) we ran a simulation (details are available upon request). The results of the simulation showed that the model was correctly able to recover all the data parameters. The final step is to use the forecasting model for production planning, using the newsvendor approach described in the earlier section. To illustrate this we will use data from a product that fits with the environment described in this study. We are in the process of acquiring this data.

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