ADDICTIVE EFFICIENCY DECOMPOSITION IN TWO-STAGE DEA

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ABSTRACT

Kao and Hwang (2008) develop a data envelopment analysis approach for measuring efficiency of two stages decision processes. The first stage uses inputs to generate outputs which are named as intermediate measures. The second stage then uses these intermediate measures to produce outputs. The current paper develops an additive efficiency decomposition approach wherein the overall efficiency is expressed as a (weighted) sum of the efficiencies of the individual stages. This approach can be applied under both constant returns to scale and variable returns to scale assumptions. The case of Taiwanese non-life insurance companies is revisited using this newly developed approach.

Keywords: data envelopment analysis (DEA), efficiency, intermediate measure, two-stage

INTRODUCTION

Data envelopment analysis (DEA) is an approach for measuring the relative efficiency of peer decision making units (DMUs) that have multiple inputs and outputs. As discussed in many DEA studies, DMUs can have a two-stage structure where the first stage uses inputs to generate outputs that then become the inputs to the second stage. The second stage thus utilizes these first-stage outputs to produce its own outputs. We call the first stage outputs intermediate measures. For example, banks use labor and assets to generate deposits which are in turn used to generate loan income (Chen and Zhu, 2004). Kao and Hwang (2008) consider a set of Taiwanese non-life insurance companies with a two-stage process of premium acquisition and profit generation. A closer examination of Kao and Hwang (2008)'s approach reveals that (i) their overall efficiency is defined as the product of efficiencies of the two stages, (ii) their models assume constant returns to scale (CRS), and (iii) their models assume that the weights (or multipliers) on the intermediate measures are the same for the two stages.

The current paper seeks to develop an alternative approach by looking at an additive combination of the efficiencies of the two stages. We assume that the overall efficiency of the two-stage process is a (weighted) sum of efficiencies of the individual stages. This additive approach enables us to study the efficiency of two-stage processes and the efficiency decomposition under the assumption of variable returns to scale (VRS). The rest of the paper is organized as follows. Sections 2 and 3 develop our alternative approach under both CRS and VRS assumptions. Section 4 applies the new approach to the 24 Taiwanese non-life insurance companies studied in Kao and Hwang (2008). Conclusions follow in Section 5.

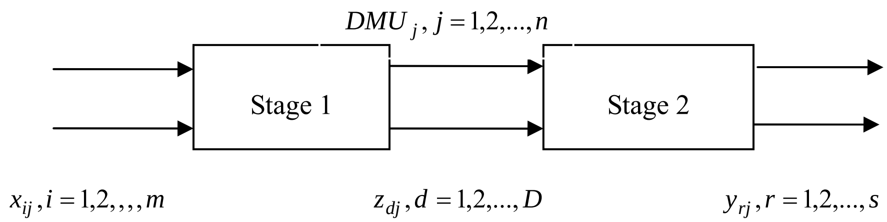
A TWO-STAGE DEA MODEL: CONSTANT RETURNS TO SCALE

Consider a two-stage process shown in Figure 1. Suppose we have n DMUs and that each DMU_j (j =1, 2, …, n) has m inputs to the first stage, x_{ij}, (i = 1, 2, …, m), and D outputs from this stage, z_{dj}, (d =1, 2, …, D). These D outputs then become the inputs to the second stage, and are referred to as intermediate measures. The outputs from the second stage are denoted y_{rj}, (r =1, 2, …, s). Based upon the CCR model (Charnes, Cooper and Rhodes, 1978), the (CRS) efficiency scores of the two-stage process and the two individual stages can be expressed as

$$\theta_j = \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \ , \quad \theta_j^1 = \sum_{d=1}^{D} \eta_d^A z_{dj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \quad \text{and} \quad \theta_j^2 = \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{d=1}^{D} \eta_d^B z_{dj} \qquad (1)$$

where v_i , η_d^A , η_d^B , and u_r are unknown non-negative weights.

Figure 1. Two-stage Process



It is useful to point out that given the individual efficiency measures θ_j^1 and θ_j^2, it is reasonable to define the efficiency of the overall two-stage process either as $\theta_j^1 \bullet \theta_j^2$ or as $w_1\,\theta_j^1 + w_2\,\theta_j^2$, where w_1 and w_2 are user-specified weights and $w_1 + w_2 = 1$. The current paper examines the performance of two-stage process when the overall efficiency of the entire process is a weighted sum of efficiencies of two individual stages, as defined in (2).

$$w_1 \cdot \left(\sum_{d=1}^{D} \eta_d^A z_{dj_0} \Big/ \sum_{i=1}^{m} v_i x_{ij_0} \right) + w_2 \cdot \left(\sum_{r=1}^{s} u_r y_{rj_0} \Big/ \sum_{d=1}^{D} \eta_d^B z_{dj_0} \right) \qquad (2)$$

We assume that $\eta_d^A = \eta_d^B (= \eta_d)$, hence we take the series relationship of the two stages into account in measuring the efficiencies. That is, we propose deriving the overall efficiency of the process by solving the problem:

$$\begin{aligned} & Max \left[w_1 \cdot \left(\sum_{d=1}^{D} \eta_d z_{dj_0} \Big/ \sum_{i=1}^{m} v_i x_{ij_0} \right) + w_2 \cdot \left(\sum_{r=1}^{s} u_r y_{rj_0} \Big/ \sum_{d=1}^{D} \eta_d z_{dj_0} \right) \right] \\ & s.t. \sum_{d=1}^{D} \eta_d z_{dj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \le 1, \quad \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{d=1}^{D} \eta_d z_{dj} \le 1 \\ & \quad \eta_d, u_r, v_i \ge 0, \ j = 1,2,...,n \end{aligned} \qquad (3)$$

It is observed that model (3) cannot be turned into a linear program using the usual Charnes-Cooper (1962) transformation. We, therefore, seek an alternative way to convert model (3) into a linear form. Letting $\sum_{i=1}^{m} v_i x_{ij_0} + \sum_{d=1}^{D} \eta_d z_{dj_0}$ represent the total size of (amount of resources consumed by) the two-stage process, and $\sum_{i=1}^{m} v_i x_{ij_0}$ and $\sum_{d=1}^{D} \eta_d z_{dj_0}$, the sizes of the stages 1 and 2 respectively, we define

$$w_1 = \sum_{i=1}^{m} v_i x_{ij_0} \Big/ \left(\sum_{i=1}^{m} v_i x_{ij_0} + \sum_{d=1}^{D} \eta_d z_{dj_0} \right) \text{and } w_2 = \sum_{d=1}^{D} \eta_d z_{dj_0} \Big/ \left(\sum_{i=1}^{m} v_i x_{ij_0} + \sum_{d=1}^{D} \eta_d z_{dj_0} \right) \quad (4)$$

Note that w_1 and w_2, as defined in the above manner, are variables related to the inputs and the intermediate measures. By virtue of the optimization process, it can turn out that either $w_1 = 1$ and $w_2 = 0$ or $w_1 = 0$ and $w_2 = 1$ at optimality. To overcome this problem, we impose that $w_1 > \alpha$ and $w_2 > \alpha$, where α is a selected constant and 0% < α < 50%. Under the constant returns to scale case, and with the imposition of the above bounds, model (3) is equivalent to the programming (5) using the Charnes-Cooper transformation.

$$\begin{aligned} & Max \sum_{r=1}^{s} \mu_r y_{rj_o} + \sum_{d=1}^{D} \pi_d z_{dj_o} \\ & s.t. \ \sum_{d=1}^{D} \pi_d z_{dj} - \sum_{i=1}^{m} \omega_i x_{ij} \le 0, \quad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \pi_d z_{dj} \le 0 \\ & \quad \sum_{i=1}^{m} \omega_i x_{ij_o} + \sum_{d=1}^{D} \pi_d z_{dj_o} = 1, \quad \sum_{i=1}^{m} \omega_i x_{ij_o} \ge \alpha, \quad \sum_{d=1}^{D} \pi_d z_{dj_o} \ge \alpha \\ & \quad \pi_d, \mu_r, \omega_i \ge 0, \ j = 1,2,...,n \end{aligned} \quad (5)$$

By changing the value of α, we can study the sensitivity of the overall efficiency scores relative to this parameter. To determine the efficiency for each stage, we propose the following procedure. As in Kao and Hwang (2008), given the overall efficiency obtained from (5) (denoted as θ_o), we calculate either the first stage's efficiency (θ_j^{1*}) or the second stage's efficiency (θ_j^{2*}) first, and then derive from that the efficiency of the other stage. In case the first stage is to be given pre-emptive priority, the following model determines its efficiency (θ_o^{1*}), while maintaining the overall efficiency score at θ_o calculated from model (5).

$$\begin{aligned} & \theta_o^{1*} = Max \sum_{d=1}^{D} \pi_d z_{dj_o} \\ & s.t. \ \sum_{d=1}^{D} \pi_d z_{dj} - \sum_{i=1}^{m} \omega_i x_{ij} \le 0, \quad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \pi_d z_{dj} \le 0 \\ & \quad (1 - \theta_o) \sum_{d=1}^{D} \pi_d z_{dj_o} + \sum_{r=1}^{s} \mu_r y_{rj_o} = \theta_o, \quad \sum_{i=1}^{m} \omega_i x_{ij_o} = 1 \\ & \quad \pi_d, \mu_r, \omega_i \ge 0, \ j = 1,2,...,n \end{aligned} \quad (6)$$

The efficiency for the second stage is then calculated as $\theta_o^2 = \left(\theta_o - w_1^* \cdot \theta_o^{1*} \right) / w_2^*$ where w_1^* and w_2^* represent optimal weights obtained from model (5) by way of (4). In case the second stage is to be given pre-emptive priority, the following model determines the second stage's efficiency (θ_o^{2*}) while maintaining the overall efficiency score at θ_o calculated from model (5).

$$\begin{aligned} & \theta_o^{2*} = Max \sum_{r=1}^{s} \mu_r y_{rj_o} \\ & s.t. \ \sum_{d=1}^{D} \pi_d z_{dj} - \sum_{i=1}^{m} \omega_i x_{ij} \le 0, \quad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \pi_d z_{dj} \le 0 \\ & \quad \sum_{d=1}^{D} \pi_d z_{dj_o} + \sum_{r=1}^{s} \mu_r y_{rj_o} - \theta_o \sum_{i=1}^{m} \omega_i x_{ij_o} = \theta_o \\ & \quad \sum_{d=1}^{D} \pi_d z_{dj_o} = 1, \quad \pi_d, \mu_r, \omega_i \ge 0, \ j = 1,2,...,n \end{aligned} \quad (7)$$

and the efficiency for the first stage is calculated as $\theta_o^1 = \left(\theta_o - w_2 \cdot \theta_o^{2*} \right) / w_1^*$.

TWO-STAGE DEA MODEL: VARIABLE RETURNS TO SCALE

While the discussion in the previous section is based upon the assumption of constant returns to scale, the above approach also enables us to study the efficiency of two-stage processes under variable returns to scale (VRS). We have the VRS overall efficiency as using the weights defined under the CRS assumption, which has the following linear programming program

$$\begin{aligned} & Max \sum_{r=1}^{s} \mu_r y_{rj_o} + u^1 + \sum_{d=1}^{D} \pi_d z_{dj_o} + u^2 \\ & s.t. \sum_{d=1}^{D} \pi_d z_{dj} - \sum_{i=1}^{m} \omega_i x_{ij} + u^1 \le 0, \quad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \pi_d z_{dj} + u^2 \le 0 \\ & \sum_{i=1}^{m} \omega_i x_{ij_o} + \sum_{d=1}^{D} \pi_d z_{dj_o} = 1, \quad \sum_{i=1}^{m} \omega_i x_{ij_o} \ge \alpha \\ & \sum_{d=1}^{D} \pi_d z_{dj_o} \ge \alpha, \quad \pi_d, \mu_r, \omega_i \ge 0, \ j = 1,2,...,n, \ u^1, u^2 \text{free in sign} \end{aligned} \tag{8}$$

Once we obtain the overall efficiency, models similar to models (6) and (7) can be developed to determine the efficiency of each stage. Specifically, assuming pre-emptive priority for stage 1, the following model determines that stage's efficiency (E_o^{1*}), while maintaining the overall efficiency score at E_o calculated from model (8).

$$\begin{aligned} & E_o^{1*} = Max \sum_{d=1}^{D} \pi_d z_{dj_o} + u^1 \\ & s.t. \sum_{d=1}^{D} \pi_d z_{dj} + u^1 - \sum_{i=1}^{m} \omega_i x_{ij} \le 0, \quad \sum_{r=1}^{s} \mu_r y_{rj} + u^2 - \sum_{d=1}^{D} \pi_d z_{dj} \le 0 \\ & (1 - E_o) \sum_{d=1}^{D} \pi_d z_{dj_o} + \sum_{r=1}^{s} \mu_r y_{rj_o} + u^1 + u^2 = E_o \\ & \sum_{i=1}^{m} \omega_i x_{ij_o} = 1, \quad \pi_d, \mu_r, \omega_i \ge 0, \ j = 1,2,...,n, \quad u^1, u^2 \text{free in sign} \end{aligned} \tag{9}$$

Similarly, if stage 2 is to e given pre-emptive priority, the following model determines the efficiency (E_j^{2*}) for that stage, while maintaining the overall efficiency score at E_o calculated from model (8).

$$\begin{aligned} & E_o^{2*} = Max \sum_{r=1}^{s} \mu_r y_{rj_o} + u^2 \\ & s.t. \sum_{d=1}^{D} \pi_d z_{dj} + u^1 - \sum_{i=1}^{m} \omega_i x_{ij} \le 0, \quad \sum_{r=1}^{s} \mu_r y_{rj} + u^2 - \sum_{d=1}^{D} \pi_d z_{dj} \le 0 \\ & \sum_{d=1}^{D} \pi_d z_{dj_o} + \sum_{r=1}^{s} \mu_r y_{rj_o} - E_o \sum_{i=1}^{m} \omega_i x_{ij_o} + u^1 + u^2 = E_o \\ & \sum_{d=1}^{D} \pi_d z_{dj_o} = 1, \ \pi_d, \mu_r, \omega_i \ge 0, \ j = 1,2,...,n, \ u^1, u^2 \text{free in sign} \end{aligned} \tag{10}$$

Once the efficiency score for one of the stages is calculated using (9) or (10), the score for the other stage can be derived in the similar manner as in the CRS case.

APPLICATION

We here apply our new approach to the 24 Taiwanese non-life insurance companies studied in Kao and Hwang (2008) (see Kao and Hwang (2008)).

The results from models (5), (6) and (7) are reported in Table 1. The results are calculated with α varying from 5% to 50% with 5% increment. Model (5) yields identical results for these different α values. Note that it is likely that model (5) can be infeasible with certain α values. For example, when α = 40%, model (5) is infeasible for DMU22 and when α= 50%, model (5) is infeasible for

DMUs 1, 2, 5, 6, 10, 16, 20 and 23. This indicates that the input mixes for these DMUs do not allow such weighting structures. Our first stage's efficiency score (θ_o^{1*}) obtained from model (6) is identical to the corresponding Kao and Hwang's (2008) score for each DMU. It can also be seen from Table 1 that we have a unique efficiency decomposition. This arises from the fact that models (6) and (7) yield identical efficiency scores for the two stages. Further, the efficiency score for our second stage is also identical to the corresponding Kao and Hwang (2008) second stage score for each DMU.

Table1. CRS Results

	CRS Overall efficiency	w_1	w_2	θ_o^{1*}	θ_o^2	θ_o^1	θ_o^{2*}
1.Taiwan Fire	0.849***	0.502	0.498	0.993	0.704	0.993	0.704
2.Chung Kuo	0.812***	0.500	0.500	0.998	0.626	0.998	0.626
3.Tai Ping	0.817**	0.592	0.408	0.690	1	0.690	1
4.China Mariners	0.596**	0.580	0.420	0.724	0.420	0.724	0.420
5.Fubon	0.873***	0.546	0.454	0.831	0.923	0.831	0.923
6.Zurich	0.689***	0.510	0.490	0.961	0.406	0.961	0.406
7.Taian	0.580**	0.571	0.429	0.752	0.352	0.752	0.352
8.Ming Tai	0.579**	0.580	0.420	0.726	0.378	0.726	0.378
9.Central	0.612	0.500	0.500	1	0.223	1	0.223
10.The First	0.713***	0.537	0.463	0.862	0.541	0.862	0.541
11.Kuo Hua	0.509**	0.578	0.422	0.729	0.207	0.729	0.207
12.Union	0.880	0.500	0.500	1	0.760	1	0.760
13.Shingkong	0.557**	0.552	0.448	0.811	0.243	0.811	0.243
14.South China	0.577**	0.580	0.420	0.725	0.374	0.725	0.374
15.Cathay Century	0.807	0.500	0.500	1	0.614	1	0.614
16.Allianz President	0.639***	0.530	0.470	0.886	0.362	0.886	0.362
17.Newa	0.613**	0.580	0.420	0.723	0.460	0.723	0.460
18.AIU	0.587**	0.558	0.442	0.794	0.326	0.794	0.326
19.North America	0.706	0.500	0.500	1	0.411	1	0.411
20.Federal	0.765***	0.517	0.483	0.933	0.586	0.933	0.586
21.Royal & Sunalliance	0.541**	0.571	0.429	0.751	0.262	0.751	0.262
22.Aisa	0.742*	0.629	0.371	0.590	1	0.590	1
23.AXA	0.685***	0.543	0.457	0.843	0.499	0.843	0.499
24.Mitsui Sumitomo	0.544	0.500	0.500	1	0.087	1	0.087

- indicates that model (6) has no feasible solution when α =40%.** indicates that model (6) has no feasible solution when α= 45%.*** indicates that model (6) will no feasible solution when α = 50%

Model (8) yields identical results on overall efficiency scores and weights for all DMUs, with the exception of DMU22. Model (8) yields a negative efficiency score when DMU 22 is under evaluation. This is due to the magnitude of the free variables in model (8). Two DMUs (5 and 22) are VRS overall efficient. Also, we have a unique VRS efficiency decomposition, as results obtained from models (9) and (10) are identical. We finally take a look at the VRS results when α equals to 45% and 50%. Table 2 reports the results when α = 45%. DMUs 21 and 22 yield negative VRS overall efficiency scores, and therefore are excluded in the calculation of each individual stage's efficiency score. Only six DMUs (3, 4, 7, 11, 14 and 17) have non-unique efficiency decompositions, namely their efficiency decompositions are different depending on whether model (9) and (10) is used. When α = 50%, only three DMUs (2, 9 and 15) yield unique efficiency decompositions. In addition to DMUs 21 and 22, DMU 23 yields a negative VRS overall efficiency score. Also, DMU4 has a negative second stage efficiency score when model (9) is used. Table 2 reports the results when α varies from 5% to 50% with 5% increment in case of VRS.

Table2. VRS Results

	When α from 5% to 40%	When α = 45%			When α = 50%		
	Overall efficiency	Overall efficiency	E_o^{1*}	E_o^{2*}	Overall efficiency	E_o^{1*}	E_o^{2*}
1.Taiwan Fire	0.867	0.867	0.990	0.743	0.867	0.995	0.743
2.Chung Kuo	0.856	0.856	1	0.711	0.855	1	0.711
3.Tai Ping	0.818	0.805	0.690	1	0.722	0.690	1
4.China Mariners	0.599	0.469	0.726	0.442	0.252	0.726	0.442
5.Fubon	1	1	1	1	0.976	1	1
6.Zurich	0.732	0.732	0.964	0.490	0.727	0.964	0.532
7.Taian	0.684	0.678	0.752	0.593	0.665	0.752	0.593
8.Ming Tai	0.754	0.754	0.783	0.722	0.742	0.816	0.722
9.Central	0.639	0.639	1	0.276	0.638	1	0.276
10.The First	0.780	0.800	0.862	0.727	0.768	0.862	0.727
11.Kuo Hua	0.614	0.594	0.740	0.458	0.545	0.741	0.458
12.Union	0.887	0.887	0.968	0.803	0.885	1	0.862
13.Shingkong	0.804	0.804	0.846	0.763	0.802	0.856	0.763
14.South China	0.654	0.622	0.725	0.560	0.568	0.725	0.562
15.Cathay Century	0.940	0.940	1	0.880	0.940	1	0.880
16.Allianz President	0.676	0.676	0.911	0.417	0.627	0.911	0.430
17.Newa	0.840	0.822	0.724	1	0.794	0.724	1
18.AIU	0.618	0.618	0.850	0.369	0.615	0.885	0.395
19.North America	0.833	0.833	1	0.657	0.824	1	0.657
20.Federal	0.946	0.946	0.902	1	0.727	1	1
21.Royal & Sunalliance	0.679	**	-	-	**	-	-
22.Aisa	1*	**	-	-	**	-	-
23.AXA	0.815	0.815	0.976	0.620	**	-	-
24.Mitsui Sumitomo	0.564	0.564	1	0.098	0.549	1	0.121

* indicates that DMU has infeasible negative VRS overall efficiency score when α = 40%.** indicates that the DMU will has negative VRS overall efficiency score.

CONCLUSIONS

The current paper adopts an alternative view of efficiency decomposition in two-stage DEA. We propose defining the overall efficiency score for a DMU as a weighted sum of the efficiencies for the individual stages, as opposed to using a simple product of those efficiencies.

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