

# TOWARD GENERATING THE EFFICIENT FRONTIERS FOR NON-CONCAVE PREFERENCES OF THE NEGOTIATORS

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## ABSTRACT

The significance of generating and accurately determining the shape and location of the frontier possibilities curve is framed both in terms of justifying post-settlement settlement (PSS) efforts and in terms of finding a fair and stable agreement to a bargaining problem. The limitations of existing approaches in generating the frontier possibilities curve of a negotiation are discussed. The paper proposes a robust method for identifying the frontier possibilities curve which captures all efficient frontier points for any combination of the negotiators' preference structures. Areas for future research are identified.

**Keywords:** negotiation analysis, frontier possibilities curve, preference structures, utility functions

## INTRODUCTION

Raiffa [14] [16], Sebenius [19], Mumpower [9], Thompson and de Harpport [24], Mumpower et al. [10] observed that the most consistent findings of decades of negotiation research are that negotiating parties fail in practice to reach efficient settlements: "Negotiators either do not settle at all, or they agree to inefficient settlements, failing to find Pareto optimal agreements" [10, p. 260]. The difference between the settlement reached and the best possible settlement is commonly referred to as the *value-left-on-the-table* and this concept has been continuously focused by negotiation analysts [7] [14] [15] [17] [18]. Figure 1 illustrates the concept of value-left-on-the-table assuming two different estimates of the frontier possibilities curve. An important challenge for negotiators is often to determine the value-left-on-the-table: after agreement  $A=(a_I, a_{II})$  is reached, is the value left-on-the-table  $v_1$  or  $v_2$ ? If  $v_1$ , one or both parties may decide to conclude the negotiations. However, if both parties realize that Frontier 2 is the actual frontier possibilities curve, they may decide to continue in the hope of realizing potential gains of up to  $v_2$ .

## THE NEGOTIATION PROBLEM STRUCTURE AND EXTREME EFFICIENT SETTLEMENTS

Negotiation researchers have greatly advanced our understanding of the structure of a negotiation problem, the characteristics of the feasible settlement space, and the shape and location of the efficient frontier. The effects of negotiation problem structure and the negotiator's strategies for reaching settlements have been the focus of several major studies since early 1990s. Sycara [21]

showed the importance of restructuring the problem under negotiation arguing that problem restructuring leads to parties' changed perceptions over the issues on the table, breaking deadlocks and increasing the parties' willingness to make concessions towards achieving improved settlement. Kersten et al. [6] utilized a sequential decision making process and analyzed the effects of restructuring the problem representation on the negotiation outcome. Mumpower [9] studied the effect of the negotiators' preference structures and their bargaining strategies on the ability to reach efficient settlements. His seminal contribution is that different combinations of the parties' utility functions determine the underlying structure of negotiation problem, that is, the shape of the efficient frontier curve. Whether the negotiators use a compromising or 'horsetrading' (i.e., log-rolling or trade-off) strategy will have opposite outcomes depending on the shape of the frontier curve. For example, a compromising strategy may result in optimal agreements when the frontier curve is concave, but it cannot produce the same result in non-concave, irregular shaped settlement spaces. Northcraft et al. [12] [13] examined the relationship between shapes of parties' marginal utility functions and negotiation outcomes. In another study, Stuhlmacher and Stevenson [20] analyzed the impact of negotiating parties' preference structures on the negotiation process in terms of their utility ratings and sequence of offers. In their empirical research, Mumpower et al. [10] studied negotiating parties' understanding of each other's payoff functions utilizing different shapes of settlement space to simulate highly integrative, integrative, fixed sum, distributive, and highly distributive experimental negotiation settings. Using a quantitative model, Vetschera [25] [26] examined the effect of the strategic manipulation of preference information by negotiating parties on the negotiation outcome.

Important studies have focused on finding efficient methods for generating efficient settlements in the last decade. Raiffa [16] proposed the critical ratio (CR) method for identifying optimal settlements in negotiations in which alternatives for each issue on the table are discrete variables or attributes. In a series of studies, Ehtamo and his colleagues introduced the constraint proposal method for generating Pareto-optimal settlements in the case of continuous decision variables [1] [2] [4] [5] [22]. The constraint proposal method was further extended to multi-party negotiations [3]. Tajima and Fraser [23] proposed "logrolling" method, an iterative quantitative trade-off approach for generating Pareto-optimal solutions in multi-issue two-party negotiations under a linear preference assumption. Using empirical data, Metcalf [8] put in perspective different methods for generating Pareto-optimal settlements in two-party negotiations and highlighted their limitations and applicability.

In this paper, the significance of generating and accurately determining the shape and location of the frontier possibilities curve is framed both in terms of justifying post-settlement settlement (PSS) efforts and in terms of finding a fair and stable agreement (i.e., an equilibrium solution) to a bargaining problem. While Mumpower and Vetschera showed the effect of different negotiating strategies and preference structures on the ability to achieve efficient outcomes, they did not offer a method for estimating the shape and location of the efficient frontier. The constraint proposal method proposed by Ehtamo and his colleagues assumed that negotiators trade off values that can be expressed by a continuous functional form, that is, the possible settlement values are continuous quantitative variables that can be mapped by continuous differentiable utility functions. In an earlier paper [11], we demonstrated that the CR method does not provide useful information about the shape or location of the frontier possibilities curve

when parties utility functions are non-concave. Addressing these limitations, we propose a method that does not make such assumptions and that is applicable to discrete variables and to any shape of the negotiators' utility functions. Our proposed method is robust and in most cases captures all efficient frontier points for any combination of the negotiators' preference structures.

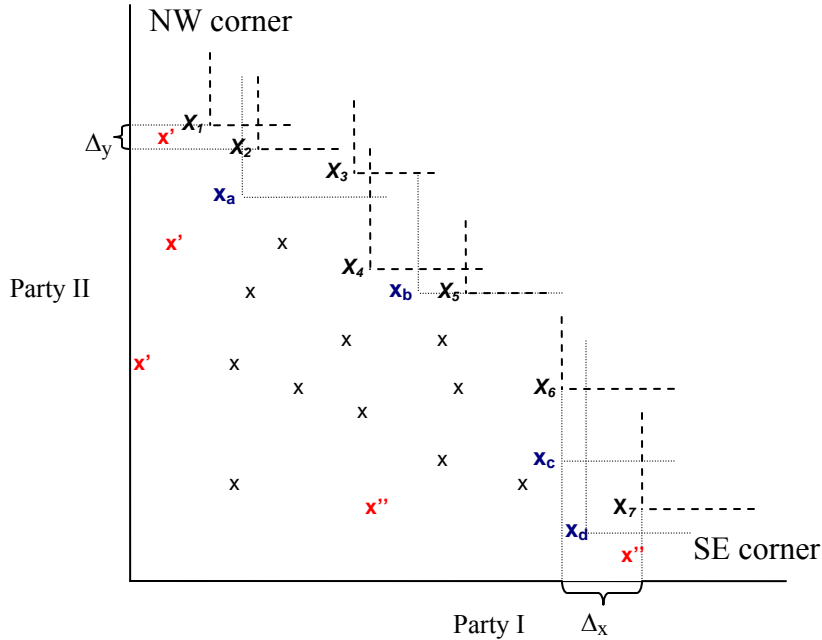
## ILLUSTRATED CASES

We have examined five different cases of preference structures of two parties negotiating over three issues, A, B, and C (see Table 1). In Case 1, all the parties' preferences are *concave* in shape, that is, preferences increase with decreasing marginal returns (increasing concave) or decrease at an accelerating rate (decreasing concave). This is the case considered in Raiffa [16]. In Case 2, the parties' preferences are *mixed* (concave preferences in two issues, and convex in the third issue) and *symmetric*, *i.e.*, both parties have the same preference structures for each issue. In Case 3, all preferences are *convex*, that is, preferences increase with increasing marginal returns (increasing convex) or decrease at a decelerating rate (decreasing convex). Cases 4 and 5 present additional combinations of *mixed asymmetric* preferences. We denote Case 5 as a highly-mixed asymmetric case since all preference structures of the parties are different for each issue. Note that Cases 4 and 5 differ from Mumpower's mixed structure cases in that he only considers the case of similar preference functions of both parties for each issue.

The results of the CR method in cases 2-5 are shown in Figure 2. The shapes of the settlement spaces (butterfly, inverted umbrella, sand dunes, and flying bat) are familiar from Mumpower (1991). The solutions identified by the CR method as frontier solutions are obviously off the mark. Case 2 is particularly noteworthy: even in a case when only one preference structure is not concave, the CR results can be quite misleading. Cases 3, 4, and 5 reinforce the conclusion that the CR method is not effective with non-concave preference structures.

## THE PROPOSED METHOD

Our proposed method overcomes the limitations of the CR method and can be effectively utilized to identify frontier solutions for all mixed preference structures of the negotiating parties. This method builds on some features of the CR method in that it also identifies a NW-most extreme-efficient solution and it uses a NW-SE direction in its mapping of the frontier possibilities curve. The method differs from the CR method in several key ways: (i) it uses the criterion of "minimum payoff increments" rather than "maximum payoff ratios"; and (ii) in addition to the NW-most solution, it also identifies the SE-most extreme-efficient solution and it also uses a SE-NW direction in its mapping. The method consists of three steps each involving a sequence of calculations: NW-SE search; SE-NW search; and mid-range zoom. The searching logic of our proposed method is illustrated in Figure 3.



**Figure 3.** Searching Logic of the Proposed Method

Starting from the NW corner, the NW-most frontier settlement is  $X_1$  (i.e.,  $f_0=X_1$ ). Let  $A(X_1) = \{X_2, X_a, X_3, \dots, X_c, \dots\}$  denote the set of candidates as frontier solutions. Only  $X_2$  is a frontier solution (i.e.,  $f_1=X_2$ ) since it provides the smallest decrease for party II and a payoff increase for party I, i.e., it is the single solution with the smallest  $\Delta y$ . Note that points such as  $X'$  were not included as candidates since they are dominated by  $X_1$ .  $X_a$  is obviously not on the frontier since  $X_2$  would provide greater payoff for both parties.  $X_3$  may or may not prove to be a frontier solution at a later point, but it will not invalidate  $X_2$  as an extreme-efficient solution, i.e., no other solution in the settlement space dominates  $X_2$ . We refer to  $X_2$  as the *least concession* by party II at  $X_1$  that party I will accept.

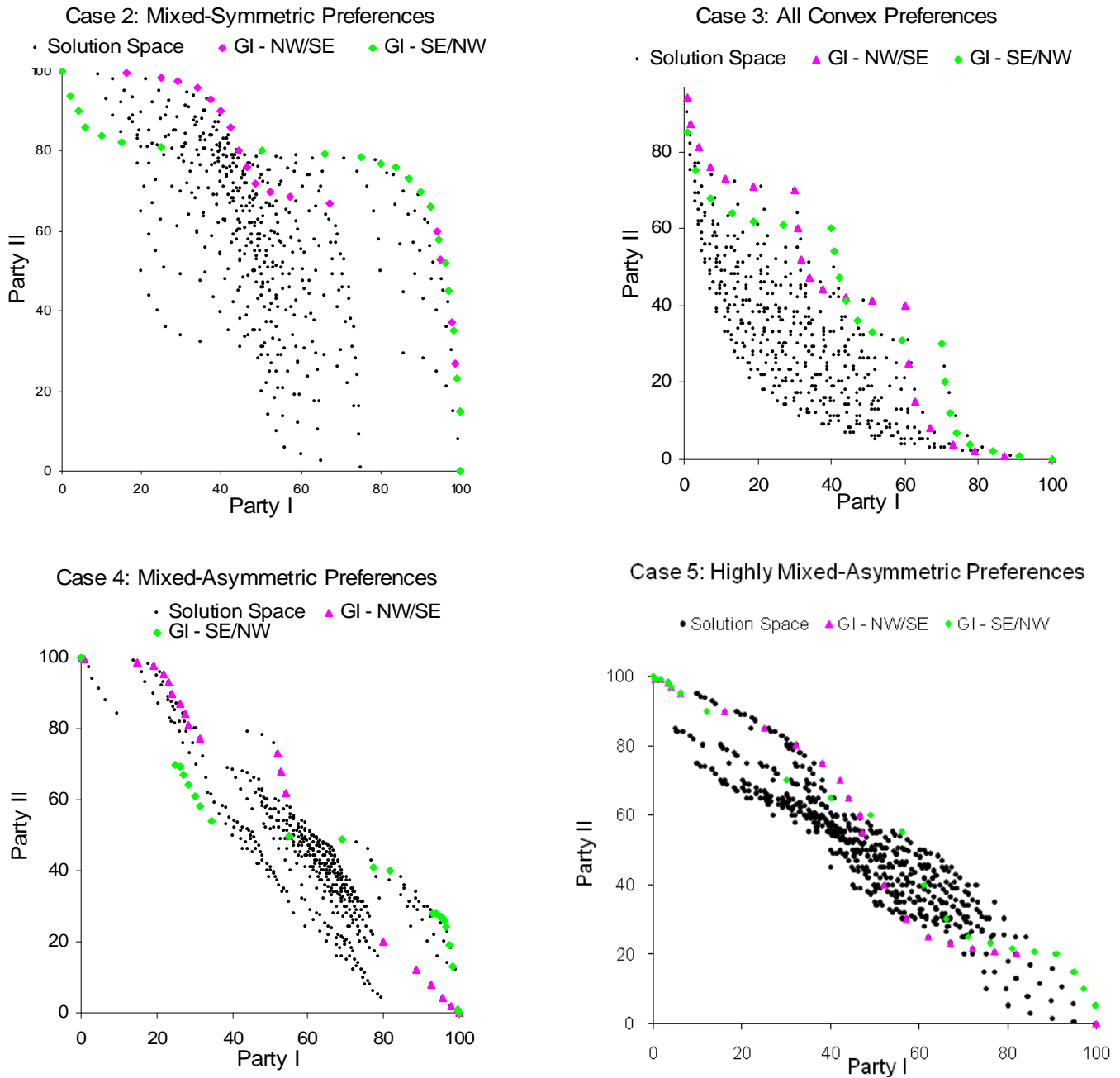
Continuing the process from  $X_2$ ,  $X_3$  is the solution with the smallest decrease for party II ( $A(X_2) = X_3$ ), and thus, indeed  $X_3$  is also an extreme-efficient solution. Moving on,  $X_4$  is also a frontier solution; but  $X_b$  is not. Both  $X_b$  and  $X_5$  represent the same decrease in payoff for party II, but obviously  $X_5$  provides higher gain for party I, and thus  $X_5$  *weakly* dominates  $X_b$ .

Applying GIM from the SE corner and moving northwesterly,  $X_7$  is clearly the SE-most extreme-efficient solution (i.e.,  $g_0=X_7$ ).  $X_d$  is the closest solution to  $X_7$ ; however, since it provides lower payoffs for both parties, it is not an extreme efficient candidate. Note that points such as  $X''$  were not included as candidates since they are dominated by  $X_7$ . Both  $X_c$  and  $X_6$  are the solutions in  $B(g_0)$  with the smallest  $\Delta x$ ; however,  $X_6$ , with greater payoff for party II, is extreme-efficient (i.e.,  $g_1=X_6$ ). Thus,  $X_6$  is the least concession by party I at  $X_7$  that party II will accept.

### THE PERFORMANCE OF THE PROPOSED METHOD

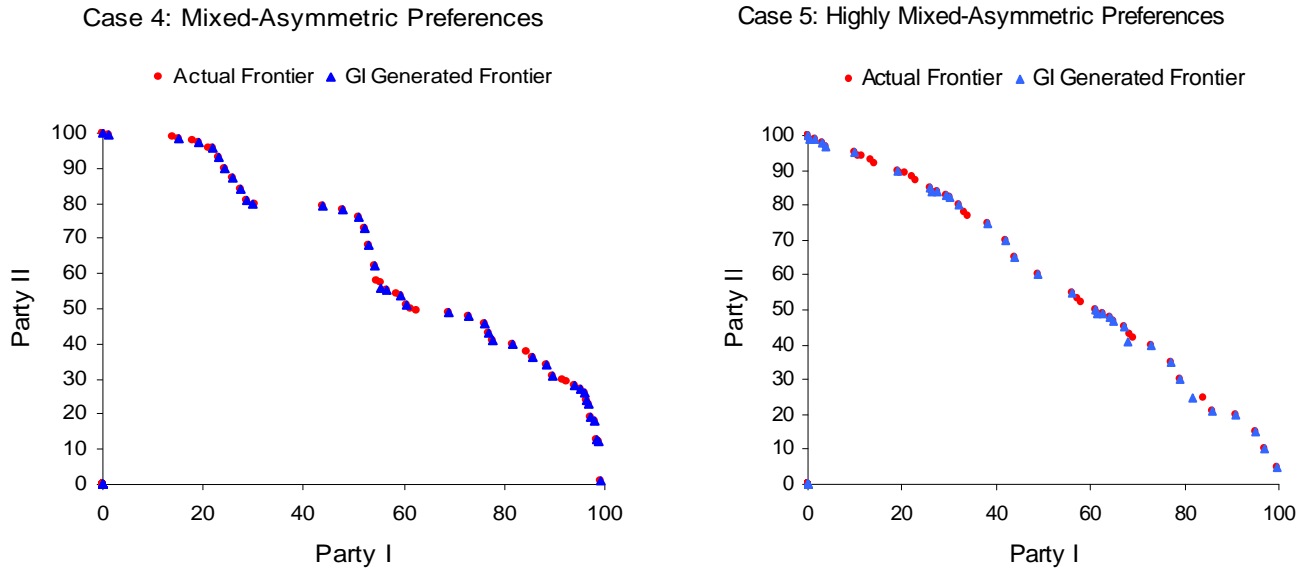
The performance of the proposed method was examined using the same five cases in Table 1. The accuracy and speed of our method depend on the preference structures of the parties. For

case 1 when all parties' preferences are concave for all issues, our method achieves effective results which are similar to those obtained by the CR method. Figure 4 shows results for cases 2-5 when not all preference structures are concave: mixed symmetric, all convex, mixed-asymmetric, and highly mixed asymmetric preferences. As shown in Fig. 4 our method generates more accurate estimates of the frontier than those provided by the CR method. We found that that in more complex cases, where highly mixed and asymmetric preference structures are present (such as cases 4 and 5), the middle range of the frontier is not perfectly captured. This can be remedied by performing the mid-range zooming procedures described above.



**Figure 4.** The Performance of GIM under Different Preference Structures: Steps 1 & 2

Figure 5 compares the actual frontier solutions with the results of our proposed method. As can be seen, it provides accurate information about the shape and location of the frontier possibilities curve.



**Figure 5.** Actual vs. Generated Frontier Solutions

## CONCLUSION

The method that we propose fills an important need in the negotiation literature: it recognizes the importance of the frontier possibilities curve and the bargaining solutions postulated by game theorists, and provides a practical method for mapping them for the benefit of the negotiating parties. We built on the research findings that plausible assumptions regarding the shape of preferences of negotiating parties may result in frontier possibilities curves that are not concave. We proposed an alternative approach for identifying frontier settlements in multi-issue two-party negotiations where alternatives are non-ordered discrete attributes. The method has a much wider application than the Constrained Proposal method since the latter method assumes continuous variables and constraints that can be expressed as continuously differentiable functions. Further, it does not make any assumptions regarding the shape of the parties' utility functions. Finally, it requires much less information disclosure and cognitive information processing than previously suggested methods.

This research can be extended beyond the multi-issue, two-party case. We agree with Metcalfe that a challenge for game and negotiation theorists is to develop effective methods for negotiations involving many parties. We are confident that our proposed method will pass the "multiparty litmus test" as called for by Metcalfe. In the complex and interrelated world in which we live, it is imperative to examine the shapes of the multi-dimensional settlement spaces when non-concave and mixed preference structures of multiple parties are present.

*Figures, table, and references are available upon request from [Behnam.Nakhai@millersville.edu](mailto:Behnam.Nakhai@millersville.edu).*