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# Traveling Salesperson String

Rick Hesse, Feature Editor

When I first encountered the traveling salesman problem, it was designated by the masculine gender, but I guess now we should call it the traveling salesperson problem (TSP). Simply stated, a salesperson must travel to *n* cities, visiting each one once and only once and ending up at the starting city to minimize cost, distance or time. This is a traveling salesperson tour, which makes a complete loop of all the nodes (cities). With the use of Excel and the built-in solver, it is fairly easy to solve tours of up to 14 cities (Hesse, 1998). The limitation of 14 is because the solver only allows up to 200 variables—the professional solver would allow for much larger problems. I have been interested lately in a related problem that I call a TSP string, which doesn't have to end up where it started, but does have to "visit"

each city. Common examples of this might be changeover costs (or setup-teardown costs) for jobs in a machine shop, where you don't have to repeat the sequence.

## Creative Student

One of my assignments for full-time working MBAs in my Quantitative Analysis class is to solve several TSP tours, including one that is a TSP string. I ask students to solve a problem of making chemicals in a tank, with given changeover costs and what would happen if they don't have to repeat the process. In other words, there is no cost for setting up for the first chemical or clean up for the last. The traditional way is to add a fictitious chemical "0" with no costs for changeover except \$999 for 0-0. Then this is solved as a tour and from the optimal solution allows the removal of the



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	A	B	C	D	E	F	G	H	I	J	K	L	
1	<b>PRINTING PRESS CHANGEOVER COSTS</b>												
2	String	COST OF CHANGEOVER											
3		1	2	3	4	5	6						
4		1	\$999	\$36	\$48	\$24	\$38	\$50					
5	F	2	\$44	\$999	\$74	\$44	\$71	\$64					
6	R	3	\$36	\$69	\$999	\$47	\$80	\$95					
7	O	4	\$50	\$30	\$38	\$999	\$75	\$95					
8	M	5	\$41	\$76	\$84	\$85	\$999	\$37					
9		6	\$53	\$67	\$107	\$87	\$42	\$999					
10		TO											
11		1	2	3	4	5	6						
12		1				1						1	
13	F	2										1	
14	R	3	1									1	
15	O	4		1								1	
16	M	5							1			1	
17		6						1				1	
18												\$169	<=Optimize
19												5	5 n-1 branches

Figure 1. First infeasible solution.

links from and to "0" to get the string. This spring semester, Jose Perdomo, an engineer at Boeing Corporation, came up with a very ingenious solution that does not require changing the size of the problem.

### Printing Press Problem

To illustrate this solution technique, let us assume that a printer has six jobs that must be run on the printing press. Each job differs in size, number of colors and the nonsymmetrical changeover costs are given in C4:H9 as shown in Figure 1. To solve as a tour, it is set up as an assignment problem. To solve as a string, it is treated as a modified assignment problem, with the setup for the solver shown in Figure 2. Also added to the setup up is "Assume Linear," "Assume Nonnegative," and the Tolerance for Integers is set to 0%. When adding the break subtour constraints, possible non-integer solutions may result, so adding the integer requirements is necessary.

I12: =SUM(C12:H12) copied to I17  
 C18:=SUM(C12:C17) copied to H18  
 I18: =SUMPRODUCT(C4:H9,C12:H17)  
 I19: =SUM(I12:I17)

By allowing each row and column in the assignment problem to be  $\leq 1$  and then the sum of the rows to be = 5, the simplex algorithm is free to choose which five of the six branches to select for a possible string. For a TSP tour of  $n$  cities there will be  $n - 1$ . In an ideal world, the initial solution will be a TSP string and thus the optimal assignment solution would also be the optimal string solution. In this problem (as in most) the original solution is not a string and graphically it looks like Figure 3 (which is not geometrically proportional).

This solution has one partial string and one subtour (a tour of  $k < n$  cities). We need to add a constraint that will break up each subtour, which thus prevents this solution from occurring again, but does not eliminate any possible strings.

### Adding Break Subtour Constraints

Each constraint that we add to the problem means that the objective function can't get better. We call these constraints break subtour constraints and this means that we

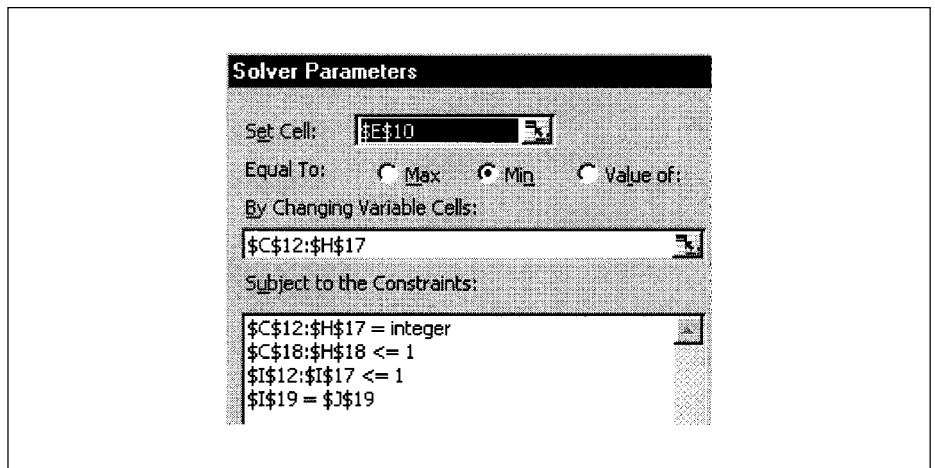


Figure 2. Solver setup.

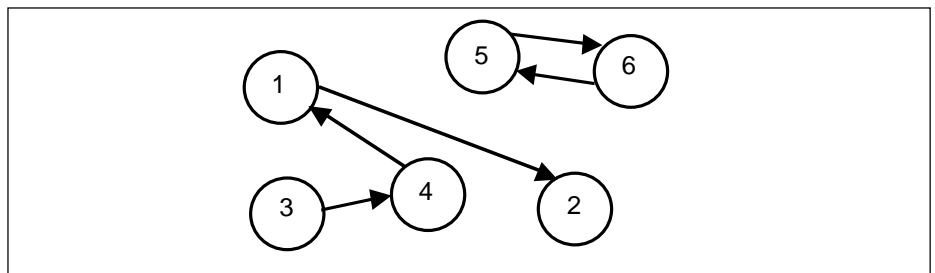


Figure 3. Initial assignment graphical solution.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	PRINTING PRESS CHANGEOVER COSTS												
2	String	COST OF CHANGEOVER											
3		1	2	3	4	5	6						
4	1	\$999	\$36	\$48	\$24	\$38	\$50						
5	F 2	\$44	\$999	\$74	\$44	\$71	\$64						
6	R 3	\$36	\$69	\$999	\$47	\$80	\$95						
7	O 4	\$50	\$30	\$38	\$999	\$75	\$95						
8	M 5	\$41	\$76	\$84	\$85	\$999	\$37						
9	6	\$53	\$67	\$107	\$87	\$42	\$999						
10		TO											
11		1	2	3	4	5	6	Break subtours	$\leq k-1$	Cost			
12	1					1		1	1	5-6-5	\$169		
13	F 2				1			1	1	2-4-2	\$185		
14	R 3	1						3	3	1-2-4-3-1	\$191		
15	O 4			1									
16	M 5							1	1				
17	6												
18								\$193	$\leq$ Optimize				
19		1	2	3	4	5	6	5	5	n-1 branches			

Figure 4. Final optimal TSP string solution.

limit each subtour of  $k$  branches to  $k - 1$  branches. Each constraint and its RHS are put on the spreadsheet in columns J and K, with an explanation and length of the incomplete solution in columns L and M. In terms of Excel, cell J12 contains H16+G17 and cell I12 the value 1, while the solver has the constraint J12<=I12 added. This means that the next solution might allow 5-6 or 6-5 or neither, but not both. Since a string can't have a tour, we have not eliminated any possible strings. After three runs, we have the solution as shown in Figure 4.

This problem required three different runs, breaking up a single subtour each time, and the cost went from \$169 to \$185 to \$191 and finally to \$193. The order of printing the jobs is 2-4-3-1-5-6 and the string graphically looks like Figure 5.

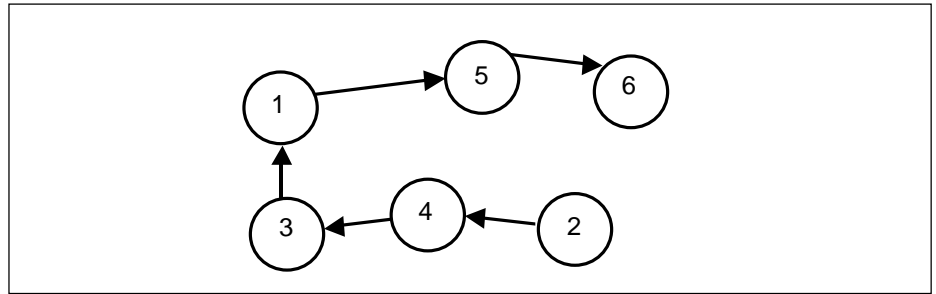


Figure 5. Optimal TSP string.

This algorithm developed by Jose shows the power of using spreadsheets and the solver to define a set of very useful problems in business and industry. It again shows the value of listening to students and their insights. ■

## Reference

Hesse, Rick. (1998). *Managerial spreadsheet modeling and analysis*. Homewood, IL: Irwin.

## BOARD OF DIRECTORS

President Lee J. Krajewski (University of Notre Dame) chaired the Board of Directors meeting that was held on April 24, 1999, in Atlanta, Georgia. The following is a report of the actions taken by the Board and matters brought to its attention. The Strategic Planning/Executive Committee also met April 22-23. Its recommendations to the Board are included in the items reported below.

1. The minutes of the January 1999 meeting of the Board of Directors were approved.
2. The financial statement for the period ended June 30, 1998, was accepted.
3. A statement of the Institute securities and cash account balances for the period ended March 31, 1999, was accepted.
4. Information and recommendations regarding the Home Office information systems were reviewed and accepted, including authorization to proceed with the selection and implementation process for new association management software for the Home Office.
5. The proposed standstill budget for FY 1999-2000 was approved in addition to the following alternative items, which become part of the approved standstill budget:
  - a. An increase to the President's Fund primarily to cover Executive

Committee and Board of Directors meeting expenses.

- b. Expense for a Continental Breakfast for the New Faculty Development Consortium.
- c. Celebratory expenses for highlighting the Institute's 30th Anniversary during the 1999 Annual Meeting.
- d. Expenses for "mementos" for the Institute's 30th Anniversary 1999 Annual Meeting attendees.
- e. Additional Annual Meeting hotel room rebate of \$3 per room per night for a total room rebate of \$10 for a total of 3,150 room nights.
6. Committee membership appointments for 1999-2000 were approved.
7. The Institute's 1999-2000 Strategic Plan with new and revised goals and initiatives was approved.
8. Charges to the 1999-2000 committees and various elected or appointed officials of the Institute were approved.
9. Objectives for the 1999-2000 Board of Directors were approved.
10. Recommendations for the reappointments of Global Development Coordinator (Robert E. Markland), and Member Services Coordinator (Scott M. Shafer) were unanimously approved.
11. The recommendation for the appointment of Vicki Smith-Daniels to serve as 2001 Annual Meeting Program Chair was unanimously approved.

12. The 1998-1999 Fellows Committee's recommendation for the selection of Kee Young Kim and Paul C. Nutt as 1999 Fellows was unanimously approved.
13. Changes to the Southeast Region's By-law 4 were approved.
14. Changes to the Western Region's Bylaws were approved.
15. The following reports were presented, reviewed and accepted:
  - a. Results of the 1999 election of officers.
  - b. Reports on submission statistics and cross-tabulated data on tracks and themes for the 1999 Annual Meeting.
  - c. Accomplishments of the 1998-99 Board of Directors.
  - d. Schedules of the 2000 Executive Committee and Board of Directors meetings.
  - e. Reports on the 1999 regional meetings.
  - f. Appointment of various coordinators for the 2000 Annual Meeting.
  - g. Results of the election of a Vice President (Mark Davis) to serve on the Executive Committee.