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Data Envelopment Analysis: Models and Extensions

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Data envelopment analysis (DEA) is receiving increasing importance as a tool for evaluating and improving the performance of manufacturing and service operations. It has been extensively applied in performance evaluation and benchmarking of schools, hospitals, bank branches, production plants, etc. (Charnes et al., 1994). This paper provides an introduction to DEA and some important methodological extensions that have improved its effectiveness as a productivity analysis tool.

DEA is a multi-factor productivity analysis model for measuring the relative efficiencies of a homogenous set of decision making units (DMUs). The efficiency score in the presence of multiple input and output factors is defined as:

$$Efficiency = \frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs} \qquad (1)$$

Assuming that there are n DMUs, each with m inputs and s outputs, the relative efficiency score of a test DMU p is obtained by solving the following model proposed by Charnes et al. (1978):

$$\begin{aligned} \max \quad & \frac{\sum_{k=1}^{s} v_k y_{kp}}{\sum_{j=1}^{m} u_j x_{jp}} \\ s.t \quad & \frac{\sum_{k=1}^{s} v_k y_{ki}}{\sum_{j=1}^{m} u_j x_{ji}} \le 1 \quad \forall i \\ & v_k, u_j \ge 0 \quad \forall k, j, \end{aligned} \qquad (2)$$

where

k = 1 to s,

j = 1 to m,

i = 1 to n,

y_{ki} = amount of output k produced by DMU i,

x_{ji} = amount of input j utilized by DMU i,

v_k = weight given to output k,

u_j = weight given to input j.

The fractional program shown as (2) can be converted to a linear program as shown in (3). For more details on model development see Charnes et al. (1978).

$$\begin{aligned} \max \quad & \sum_{k=1}^{s} v_k y_{kp} \\ s.t \quad & \sum_{j=1}^{m} u_j x_{jp} = 1 \\ & \sum_{k=1}^{s} v_k y_{ki} - \sum_{j=1}^{m} u_j x_{ji} \le 0 \quad \forall i \\ & v_k, u_j \ge 0 \quad \forall k, j. \end{aligned} \qquad (3)$$

The above problem is run n times in identifying the relative efficiency scores of all the DMUs. Each DMU selects input and output weights that maximize its efficiency score. In general, a DMU is considered to be efficient if it obtains a score of 1 and a score of less than 1 implies that it is inefficient.

Benchmarking in DEA

For every inefficient DMU, DEA identifies a set of corresponding efficient units that can be utilized as benchmarks for improvement. The benchmarks can be obtained from the dual problem shown as (4).

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$$\min \quad \theta$$

$$s.t \quad \sum_{i=1}^{n} \lambda_i x_{ji} - \theta x_{jp} \leq 0 \quad \forall j$$

$$\sum_{i=1}^{n} \lambda_i y_{ki} - y_{kp} \geq 0 \quad \forall k$$

$$\lambda_i \geq 0 \quad \forall i \tag{4}$$

where

θ = efficiency score, and

λs = dual variables.

Based on problem (4), a test DMU is inefficient if a composite DMU (linear combination of units in the set) can be identified which utilizes less input than the test DMU while maintaining at least the same output levels. The units involved in the construction of the composite DMU can be utilized as benchmarks for improving the inefficient test DMU. DEA also allows for computing the necessary improvements required in the inefficient unit's inputs and outputs to make it efficient. It should be noted that DEA is primarily a diagnostic tool and does not prescribe any reengineering strategies to make inefficient units efficient. Such improvement strategies must be studied and implemented by managers by understanding the operations of the efficient units.

Although benchmarking in DEA allows for the identification of targets for improvements, it has certain limitations. A difficulty addressed in the literature regarding this process is that an inefficient DMU and its benchmarks may not be inherently similar in their operating practices. This is primarily due to the fact that the composite DMU that dominates the inefficient DMU does not exist in reality. To overcome these problems researchers have utilized performance-based clustering methods for identifying more appropriate benchmarks (Doyle & Green, 1994; Talluri & Sarkis, 1997). These methods cluster inherently similar DMUs into groups, and the best performer in a particular cluster is utilized as a benchmark by other DMUs in the same cluster.

Performance Ranking in DEA

Traditional DEA models do not allow for ranking DMUs, specifically the efficient ones. Also, it is possible in DEA that some of the inefficient DMUs are in fact better overall performers than certain efficient ones. This is because of the unrestricted weight flexibility problem in DEA. In the determination of relative efficiency, problem (3) allows for unrestricted factor weights (v_k and u_j). Thus, a DMU can achieve a high relative efficiency score by being involved in an unreasonable weight scheme (Dyson & Thannassoulis, 1988; Wong & Beasley, 1990). Such DMUs heavily weigh few favorable measures and completely ignore other inputs and outputs. These DMUs can be considered as niche members and are not good overall performers. Cross-efficiencies in DEA is one method that could be utilized to identify good overall performers and effectively rank DMUs (Sexton et al., 1986).

This paper provides an introduction to DEA and some important methodological extensions that have improved its effectiveness as a productivity analysis tool.

Cross-efficiency methods evaluate the performance of a DMU with respect to the optimal input and output weights (v_k and u_j) of other DMUs. The resulting evaluations can be aggregated in a cross-efficiency matrix (CEM). In the CEM, the element in *i*th row and *j*th column represents the efficiency of DMU *j* when evaluated with respect to the optimal weights of DMU *i*. A DMU, which is a good overall performer, should have several high cross-efficiency scores along its column in the CEM. On the other hand, a poorly performing DMU should have several low values. The column means can be computed to effectively differentiate between good and poor performers (Boussofiane et. al, 1991).

A limitation in using the CEM is that the factor weights obtained from problem (3) may not be unique. This undermines the effectiveness of the CEM in discriminating between good and poor performers. Some techniques have been proposed for obtaining robust factor weights for use in the construction of the CEM. Doyle and Green (1994) have developed a set of formulations for this purpose. The one that is most appropriate for this discussion is the aggressive formulation, which identifies optimal weights that not only maximize the efficiency of a unit but also minimize the efficiency of the average unit that is constructed from the other $n - 1$ units. For more information on the development and applicability of this and other related models, see Doyle and Green (1994).

Talluri (2000) proposed a variation to the Doyle and Green model, which compares a pair of DMUs each time. In this model, the target DMU (evaluator) not only maximizes its efficiency score but also minimizes the efficiency score of each competitor, in turn. Therefore, the optimal weights of the target DMU may vary depending on the competitor being evaluated. In essence, the target DMU can involve multiple strategies (optimal solutions or the input and output weights), that is, it emphasizes its strengths, which are weaknesses of a specific competitor. These results can be incorporated into a CEM to identify good overall performers.

Sarkis and Talluri (1999) extended the above case to include both cardinal and ordinal input and output factors, which is based on the work by Cook et al. (1996). They proposed a combination of models that allowed for effective ranking of DMUs in the presence of both quantitative as well as qualitative factors. These models are also based on cross-evaluations in DEA.

Other ranking methods that do not specifically include cross-efficiencies were proposed by Rousseau and Semple (1995), and Andersen and Petersen (1993). Rousseau and Semple (1995) approached the same problem as a two-person ratio efficiency game. Their formulation provides a unique set of weights in a single phase as opposed to the two-phase approaches presented above. Andersen and Petersen (1993) proposed a ranking model, which is a revised version of problem (3). In this model, the test DMU is removed from the constraint set allowing the DMU to achieve an efficiency score of greater than 1, which provides a method for ranking efficient and inefficient units.

Weight Restrictions in DEA

As mentioned in the previous section, problem (3) allows for unrestricted weight flexibility in determining the efficiency scores of DMUs. This allows units to achieve relatively high efficiency scores by indulging in inappropriate input and output factor weights. Weight restrictions allow for the integration of managerial preferences in terms of relative importance levels of various inputs and outputs. For example, if output 1 is at least twice as important as output 2 then this can be incorporated into the DEA model by using the linear constraint $v_1 \geq 2v_2$. Methods for incorporating weight restrictions have been suggested by several researchers. Included in this stream of research are works by Charnes et al. (1990), Dyson and Thannassoulis (1988), Thompson et al. (1986, 1990, 1995), and Wong and Beasley (1990). Although weight restrictions effectively discriminate between efficient and inefficient units, ranking DMUs can still be an issue. In order to allow for a ranking of units in the presence of weight restrictions, a combination of models proposed by Talluri and Yoon (2000) could be utilized.

Efficiency Changes Over Time

In order to capture the variations of efficiency over time, Charnes et al. (1985) proposed a technique called 'window analysis' in DEA. Window analysis assesses the performance of a DMU over time by treating it as a different entity in each time period. This method allows for tracking the performance of a unit or a process. For example, if there are *n* units with data on their input and output measures in *k* periods, then a total of *nk* units need to be assessed simultaneously to capture the efficiency variations over time.

In the traditional window analysis described above, when a new period is introduced into the window the earliest period is dropped out. A variation to this method was proposed by Talluri et al. (1997) to effectively monitor the performance of a unit over time and assist in process improvement and benchmarking. Essentially, this technique, referred to as the 'modified window analysis,' drops the poorest performing period instead of the earliest period. This allows for a new period to be challenged against the best of the previous periods and, thereby, assisting in process improvement and benchmarking.

Other DEA Models

The models discussed so far in this paper work under the assumption of constant returns to scale. While this is often a legitimate assumption, in situations where constant returns to scale do not prevail it is important to compare units based on their scale of operations. Banker et al. (1984) proposed a model that can be used under conditions of non-constant returns to scale.

DEA has also been utilized as a resource allocation tool. A good example of its use in resource allocation can be found in Bessent et al. (1983). The incorporation of categorical variables into DEA evaluations can be found in Banker and Morey (1986) and Kamakura (1988). Some work in the consideration of both cardinal and ordinal factors in DEA can be found in Cook et al. (1993, 1996), Sarkis and Talluri (1999).

Conclusions

In this paper we have introduced DEA and some methodological extensions that could be utilized to improve its discriminatory power in performance evaluation. The primary advantages of this technique are that it considers multiple factors and does not require parametric assumptions of traditional multivariate methods. However, there are some critical factors one must consider in the application of DEA models. The efficiency scores could be very sensitive to changes in the data and depend heavily on the number and type of input and output factors considered. In general, inputs can include any resources utilized by a DMU, and the outputs can range from actual products produced to a range of performance and activity measures. The size of the data set is also an important factor when using some of the traditional DEA models. As a general rule, with five inputs and five outputs, at least 25 or so units will appear efficient and so the set needs to be much greater than 25 for any discrimination. However, some of these sample size problems can be overcome by using cross-efficiency models discussed in this paper.

The review in this paper is in no way exhaustive of developments in this field. For example, there is significant work in the areas of stochastic DEA, profiling in DEA, sensitivity analysis in DEA, target setting in DEA, more effective ways of weight restrictions in DEA, among other developments. Some of the interesting extensions in this area can include the improvement of discriminatory power of non-constant returns to scale models, better methods for benchmarking, developing the robustness of cross-efficiency models, etc. Each of these new models and methods can be useful in a variety of manufacturing and service areas.

References

Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, *39*(10), 1261-1264.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science*, *30*(9), 1078-1092.

Banker, R. D., & Morey, R. (1986). Efficiency analysis for exogenously fixed inputs and outputs. *Operations Research*, *34*(4), 513-521.

Bessent, A., Bessent, W., Charnes, A., Cooper, W. W., & Thorogood, N. (1983). Evaluation of educational program proposals by means of data envelopment analysis. *Educational Administration Quarterly*, *19*(2), 82-107.

Boussofiane, A., Dyson, R. G., & Thanassoulis, E. (1991). Applied data envelopment analysis. *European Journal of Operational Research*, *52*, 1-15.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429-444.

Charnes, A., Clark, T., Cooper W. W., & Golany, B. (1985). A developmental study of data envelopment analysis in measuring the efficiency of maintenance units in U. S. Air Forces. In R. Thompson & R. M. Thrall (Eds.), *Annals of Operational Research*, *2*, 95-112.

Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1990). Polyhedral cone-ratio DEA models with an illustrative application to large commercial banks. *Journal of Econometrics*, *30*, 91-107.

Charnes, A., Cooper, W. W., Lewin, A. Y., & Seiford, L. M. (Eds.). (1994). Data envelopment analysis: Theory, methodology, and applications. Boston: Kluwer.

Cook, W. D., Kress, M., & Seiford, L. M. (1993). On the use of ordinal data in data envelopment analysis. *Journal of the Operational Research Society*, *44*, 133-140.

Cook, W.D., Kress, M., & Seiford, L.M. (1996). Data envelopment analysis in the presence of both quantitative and qualitative factors. *Journal of the Operational Research Society*, *47*(7), 945-953.

Doyle, J., & Green, R. (1994). Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. *Journal of the Operational Research Society*, *45*(5), 567-578.

Dyson, R. G., & Thannassoulis, E. (1988). Reducing weight flexibility in data envelopment analysis. *Journal of Operational Research Society*, *39*(6), 563-576.

Kamakura, W. A. (1988). A note on the use of categorical variables in data envelopment analysis. *Management Science*, *34*(10), 1273-1276.

Rousseau, J. J., & Semple, J. H. (1995). Two-person ratio efficiency games. *Management Science*, *41*(3), 435-441.

Sarkis, J., & Talluri, S. (1999). A decision model for evaluation of flexible manufacturing systems in the presence of both cardinal and ordinal factors. *International Journal of Production Research*, *37*(13), 2927-2938.

Sexton, T. R., Silkman, R. H., & Hogan, A. (1986). Data envelopment analysis: Critique and extensions. In R. H. Silkman (Ed.), *Measuring efficiency: An assessment of data envelopment analysis.* Publication no. 32 in the series New Directions of Program Evaluation, Jossey Bass, San Francisco.

Talluri, S., & Sarkis, J. (1997). Extensions in efficiency measurement of alternate machine component grouping solutions via data envelopment analysis. *IEEE Transactions on Engineering Management*, *44*(3), 299-304.

Talluri, S., Huq, F., & Pinney, W. E. (1997). Application of data envelopment analysis for cell performance evaluation and process improvement in cellular manufacturing. *International Journal of Production Research*, *35*(8), 2157-2170.

Talluri, S. (forthcoming, 2000). A benchmarking method for business process reengineering and improvement. *International Journal of Flexible Manufacturing Systems*.

Talluri, S., & Yoon, K. P. (forthcoming, 2000). A cone-ratio DEA approach for AMT justification. *International Journal of Production Economics*.

Thompson, R. G., Singleton, F. D., Thrall, R. M., & Smith, B. A. (1986). Comparative site evaluations for locating a high energy physics laboratory in Texas. *Interfaces*, *16*, 35-49.

Thompson, R. G., Langemeier, L. N., Lee, C. T., & Thrall, R. M. (1990). The role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics*, *46*(1/2), 93-108.

Thompson, R. G., Dharmapala, P. S., & Thrall, R. M. (1995). Linked-cone DEA profit ratios and technical efficiency with application to Illinois coal mines. *International Journal of Production Economics*, *39*, 99-115.

Wong, Y. H B., & Beasley, J. E. (1990). Restricting weight flexibility in data envelopment analysis. *Journal of the Operational Research Society*, *41*(9), 829-835. ■

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NAMES IN THE NEWS

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Krishna S. Dhir, Pennsylvania State University at Harrisburg, was awarded the 2000 Excellence in Teaching Award by the provost of the university in March 2000. This competitive award is given annually to a faculty member at the university who has demonstrated sustained excellence and lasting impact in the areas of general teaching, enthusiasm, commitment, academic advising, and overall career guidance to the students. Dr. Dhir also received the Best Applications Paper Award at the DSI International Conference in Athens, Greece, in July 1999. He was awarded the Best Theoretical/Empirical Research Paper award at the 1993 DSI Annual Meeting in Washington, DC.

Joseph Sarkis, Clark University, has been awarded one of the 1999 Reviewer Excellence Awards by the *Journal of Operations Management*, the E&R Foundation, and Elsevier Science. This award is based on quality of review, timeliness of responses, volume of reviews, and reviewing consistency criteria.

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