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Assignment Models and Degenerate Reports

Rick Hesse, Feature Editor

There is a problem with linear programming involving sensitivity analysis for models that are degenerate which can be illustrated by a simple assignment model. The basic problem is that these reports may "lie" in that the sensitivity ranges on the objective coefficient may be too narrow or that the marginal values may be incorrect.

Assignment Model

Figure 1 shows the spreadsheet assignment model with the hours for each crew to complete each possible job given in B4:E7 and the optimal assignments in B10:E13. Each row sums up to 1 (each job must be done once and only once-F10:F13) and each column sums up to 1 (each crew must do one job and one job only-B14:E14). Conditional formatting shades the hours for the assignment. The minimum total is 47 hours. The zeros have been hidden (under Tools ... Options, uncheck "Zero values").

The cell formulas of interest are:

F10: = SUM(B10:E10) and copy to
F11: F13

B14: = SUM(B10:B13) and copy to
C14: E14

F14: = SUMPRODUCT(B4:E7, B10:E13)

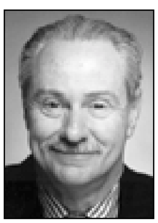
The Solver setup is shown in Figure 2.

Degeneracy in Sensitivity Report

This solution is extremely degenerate because each of the eight constraints is binding (being equalities) and yet only four variables are greater than zero (the four assignments). We will not be interested in the bottom part (constraints section) of the Excel sensitivity report because all the constraints are equalities and the marginal values (MVs) are meaningless. (This is because the interpretation of the MVs is the effect on the objective function by adding one

	A	B	C	D	E	F	G
1	LINE CREW ASSIGNMENT						
2	CREW1						
3	HOURS	CREW A	CREW B	CREW C	CREW D		
4	JOB 1	11	17	8	16		
5	JOB 2	9	7	12	6		
6	JOB 3	13	16	15	12		
7	JOB 4	21	24	17	28		
8							
9	ASSIGN	CREW A	CREW B	CREW C	CREW D		
10	JOB 1	1				1	
11	JOB 2		1			1	
12	JOB 3				1	1	
13	JOB 4			1		1	
14		1	1	1	1	47	<== Min

Figure 1: Data and network representation of project.



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unit to each particular RHS while the others remain constant. This would make the model infeasible and thus the answers meaningless.)

Just the first three rows of the Excel sensitivity report are shown in Figure 3, as well as the easier-to-read postoptimal report written from it. The first line looks just fine. If the hours for CREWA to do JOB1 increases from 11 to 12 (or 12.0001), we will have an alternate solution; if we decrease the hours for JOB1-CREWB to 15 (or 14.9999), the assignment should enter the solution. And, in fact, rerunning the solver for these values confirms that the first line is correct. We will look at the second row momentarily. However, the third row of the sensitivity and postoptimal report makes no sense. JOB1-CREWC is not assigned, and yet there is a finite upper limit of 10 hours. The only possible interpretation is that if the time for this assignment exceeds the upper bound, the solution should change. But the only possible change would be to go from no assignment (0) to assigning it (1). This is clearly impossible. Many of the variables in the sensitivity report have both finite upper and lower bounds, which is a sign that something is wrong with the sensitivity information. Clearly, for each variable with no assignment (0) in the optimal solution, the upper bound should be 1E+30 (infinity). And in similar fashion, for each variable with an assignment (1) in the optimal solution, the lower bound should be -1E+30.

On first glance, the second line makes sense. At 17 hours, it is too time-consuming for JOB1 to be done by CREWB, and certainly anything higher will do nothing to make it attractive for that assignment to reduce the total hours—so the 1E+30 upper bound makes sense. But the computed lower limit is 15 hours and the marginal value is also in agreement, stating that if we forced that assignment to equal 1, then the total cost would increase by 2. When we set C15 on the model sheet to 14.999 and rerun the solver, we get the current solution instead of a new solution. Now when we get the new sensitivity and postoptimal report, the first three rows of

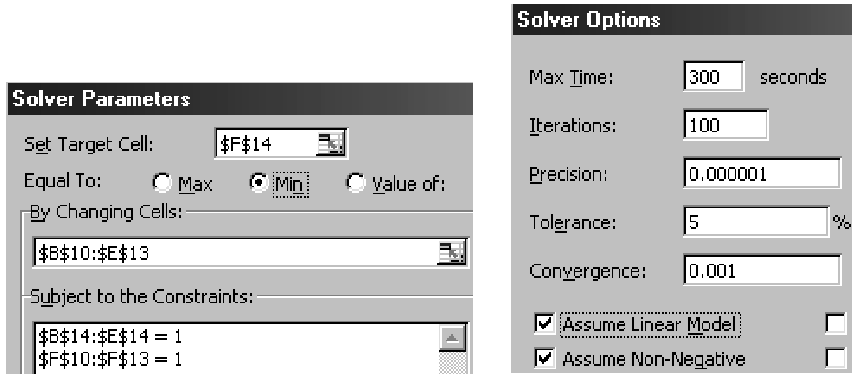

Figure 2: Solver setup for assignment model.

Excel Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B11	JOB 1 CREW A	1		11	1	1E+30
C11	JOB 1 CREW B		2	17	1E+30	2
D11	JOB 1 CREW C			8	2	1

Postoptimal Report

Decision Name	Final Value	Marginal Value	Lower Limit	Original Value	Upper Limit
JOB 1 CREW A	1		-1E+30	11	12
JOB 1 CREW B		2	15	17	1E+30
JOB 1 CREW C			7	8	10

Figure 3: Excel sensitivity report and resulting postoptimal report.

Postoptimal Report

Decision Name	Final Value	Marginal Value	Lower Limit	Original Value	Upper Limit
JOB 1 CREW A	1		-1E+30	11	12
JOB 1 CREW B			11	15	15
JOB 1 CREW C			8	8	17

Figure 4: New postoptimal report.

Crew B	Time	1-B
Job 1	47	
16.999	47	
15.999	47	
14.999	47	
13.999	47	
12.999	47	
11.999	47	
10.999	47	1
9.999	46	1
8.999	45	1
7.999	44	1

Figure 5: Solver table for changing crew hours.

the postoptimal report give us some new limits, as shown in Figure 4.

The report now states that if the number of hours for JOB1-CREWB is lowered to 11, then an assignment will be made for CREWB to do JOB1. In fact, making up a Solver Table, shown in Figure 5, shows that the true lower limit is indeed 11. (Instead of using integer hours, a value a little below each integer is used so that the change in assignment will be detected instead of the current solution, which would be an alternate optimal solution.)

The Mathematics of Degeneracy

The mathematical reason for this false sensitivity report is the extreme degree of degeneracy. If we can measure the degree of degeneracy by the number of binding constraints minus the number of variables greater than zero, then we have 4 degrees of degeneracy (0 degrees would be for a non-degenerate solution). This means that for this model, every optimal solution point is defined by eight binding constraints instead of four. Bounds in the sensitivity report are computed in the simplex method by computing how much the objective coefficient can be tilted by increasing or decreasing one particular objective coefficient before the current solution is no longer optimal—that is, the new optimal point is defined by another constraint replacing one of the binding constraints. This leads to some false signals. It would be impossible to tilt past a constraint, but the "tilting" can stop early at a constraint that doesn't change the value of the variables. In n-space, these optimal points are the same and overlay each other. For those of you who understand the mathematics of the simplex method, one zero that was in the basis is now out, and one that was out is now in, thus giving the same solution (in or out of the basis, both variables are equal to zero) but the solution different set of sensitivity ranges. Because of this degeneracy, the ranges in the postoptimal report may be correct or too narrow, but never too wide.

Conclusion

As can be seen from this simple example and analysis, the range of the objective coefficient for assignment problems will never be larger than it should be (too high, too low, or both). Another way of saying this is that the range limits will be at least what has been calculated. The same phenomena takes place in transportation problems and network flow models, and care needs to be taken when interpreting the postoptimal results for all of these degenerate models. It is recommended that for data which are in dispute, run a Solver Table for each one with values wider than the sensitivity range to check to see if these are indeed the true ranges. ■

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